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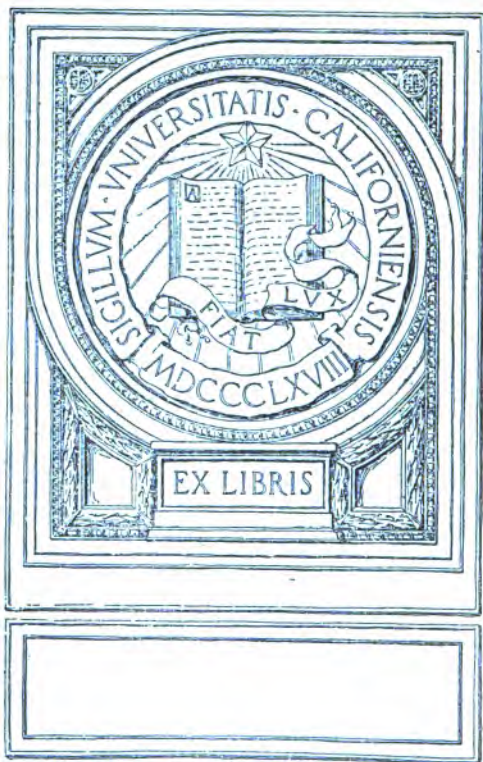
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UNIV. OF
CALIFORNIA

INSTRUCTIONS

IN

PRACTICAL SURVEYING;

TOPOGRAPHICAL PLAN DRAWING.

Black

AND ON

SKETCHING GROUND WITHOUT INSTRUMENTS.

With Plates and Woodcuts.

By G. D. BURR, PROF.



SECOND EDITION.

LONDON:
JOHN MURRAY, ALBEMARLE STREET.
1847.

TO VNU
ABROGLAO

TAS-15
BE

TO
LIEUTENANT-GENERAL
SIR GEORGE SCOVELL,
K.C.B.
&c. &c. &c.
GOVERNOR OF THE ROYAL MILITARY COLLEGE,

This Book
IS MOST RESPECTFULLY INSCRIBED,
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GEO. D. BURR.

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P R E F A C E

TO

THE SECOND EDITION.

THE first edition of this work originated in the wants of the Gentlemen Cadets in 1813 and 1814: more was added as it became requisite, the advantage of which was very apparent, as from each reading the manuscript their instruction was rendered more uniform, and much time was saved by this means. An officer who had not studied at the Senior Department complaining to the author of the inconvenience he experienced when having occasion to find the latitude, variation of the needle, &c., abroad, with such instruments as he could get, and without any previous knowledge of the subject, a little was added on practical astronomy, not by any means with an intention of substituting it for the more correct observations and rules of other works. The first publication came out, after numerous solicitations, in 1839.

In the present edition much has been added that may be useful. In the former, levelling was not very fully explained: but it has now become so necessary to many persons, and the methods and instruments are so much improved, as to require further illustration: this has accordingly been done.

Royal Military College.

P R E F A C E

TO

THE FIRST EDITION.

THE following pages are intended to facilitate the acquisition of an art now becoming daily more important; and it is hoped that the student who is well grounded in the elementary parts of mathematics will find himself provided with a complete set of instructions in the following work.

The nature of the work being entirely practical, and not entering into those niceties which characterise geodesical measurements, we have not begun with them, but beg leave to recommend the student to a perusal of the elaborate and highly interesting works, both English and foreign, upon that subject. Our object being to take up the business where geodesia* ends, requires only a good application of more simple instruments; yet as a survey or sketch may sometimes be required of sufficient extent to render triangulation necessary, we have given some practical hints upon it.

* Some formulæ upon this subject will be found at the end, by Mr. G. W. Hearn, B. A., of Cambridge, which he has kindly supplied.

It was originally intended only to form a set of instructions for military sketching; but upon consideration it was thought better to begin by surveying, as the principles of the former are derived from the latter, and the work would thereby become more generally useful.

From a conviction that the mere writings of the closet only tend to deceive the pupil, we have inserted original examples; thinking it better to state the thing fairly, than to use such as are constructed on purpose to agree with mathematical solutions. We desire to show what may be expected, and to guard the pupil against a very common supposition, namely, that when he is provided with instruments and books his work is half finished.

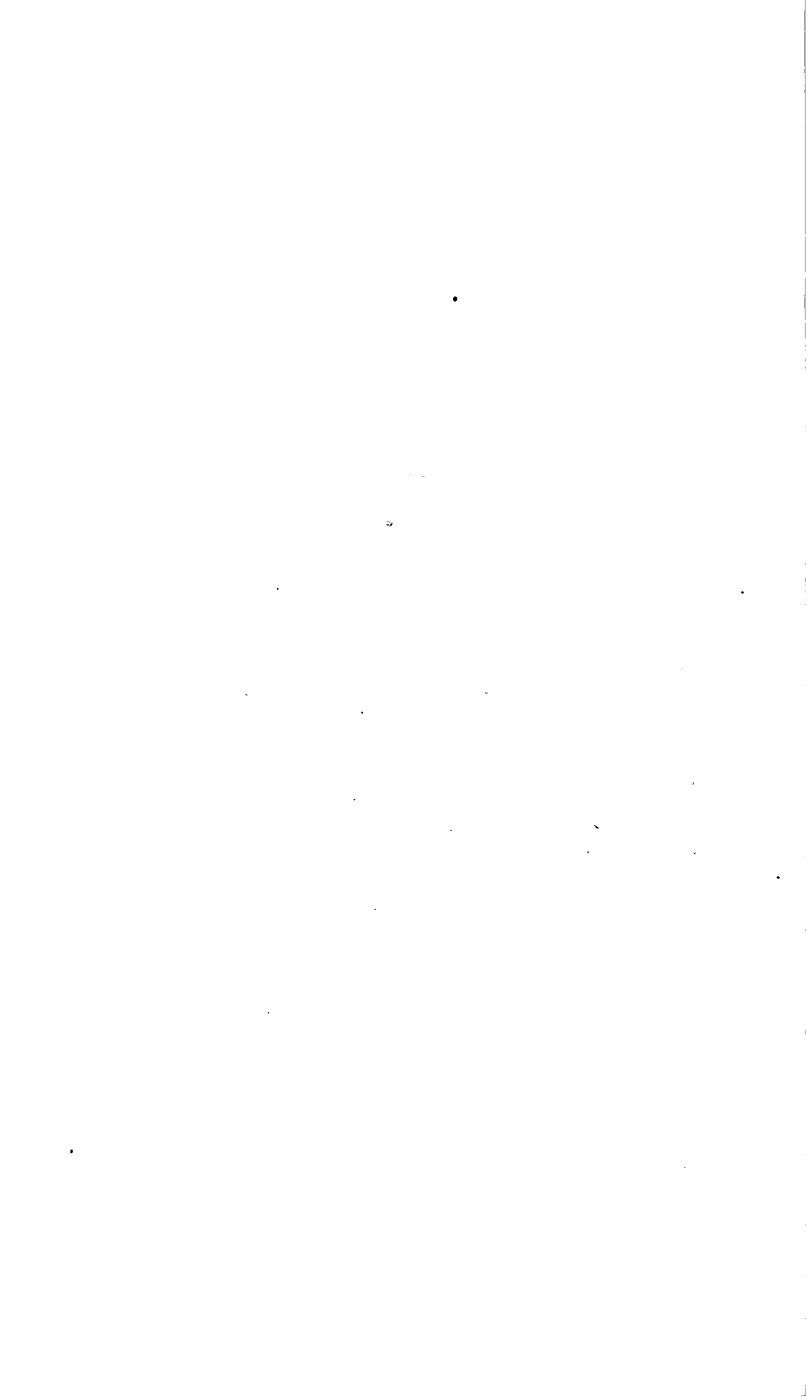
The various tables to be used in the geographical approximations are contained in the Nautical Almanack and other works, indispensably necessary to a traveller who intends to collect them.

It is a matter of much regret that we cannot produce drawings of a complete set of specimens of the great geological features of the earth; but they are chiefly contained in plans carefully lodged in the different government depôts, where, being connected with fortification, or other national works, it would be highly improper to submit them to indiscriminate inspection: besides, the geological character has hitherto been so little attended to, or so

much disfigured by more attention to style of representation than to truth of expression, that we doubt if the thing could be accomplished at the present time. We must not, however, omit to mention the Isle of Elba, and some parts of Switzerland, by the French engineers and others, which go very far to establish a mode of expressing mountains naturally.

We have given a table, and a plate of military details; but we may remark that these minutiae are, for the most part, only a diminutive representation of the real object; and in omitting field-works, &c. we refer the reader to the books on permanent and field fortification, where he will find ample information on that head.

We do not hope to give information to the old and experienced military surveyor and draftsman; but address this work to the young and uninitiated student, confidently recommending to him a practice founded upon long experience, and certain in its results, within the limits we have assigned to it. Beyond these, more capital instruments, and greater mathematical knowledge than is to be found in this work, must be called in to his assistance; but on the higher branches of surveying excellent works already exist, and few persons are ever called upon to perform them.



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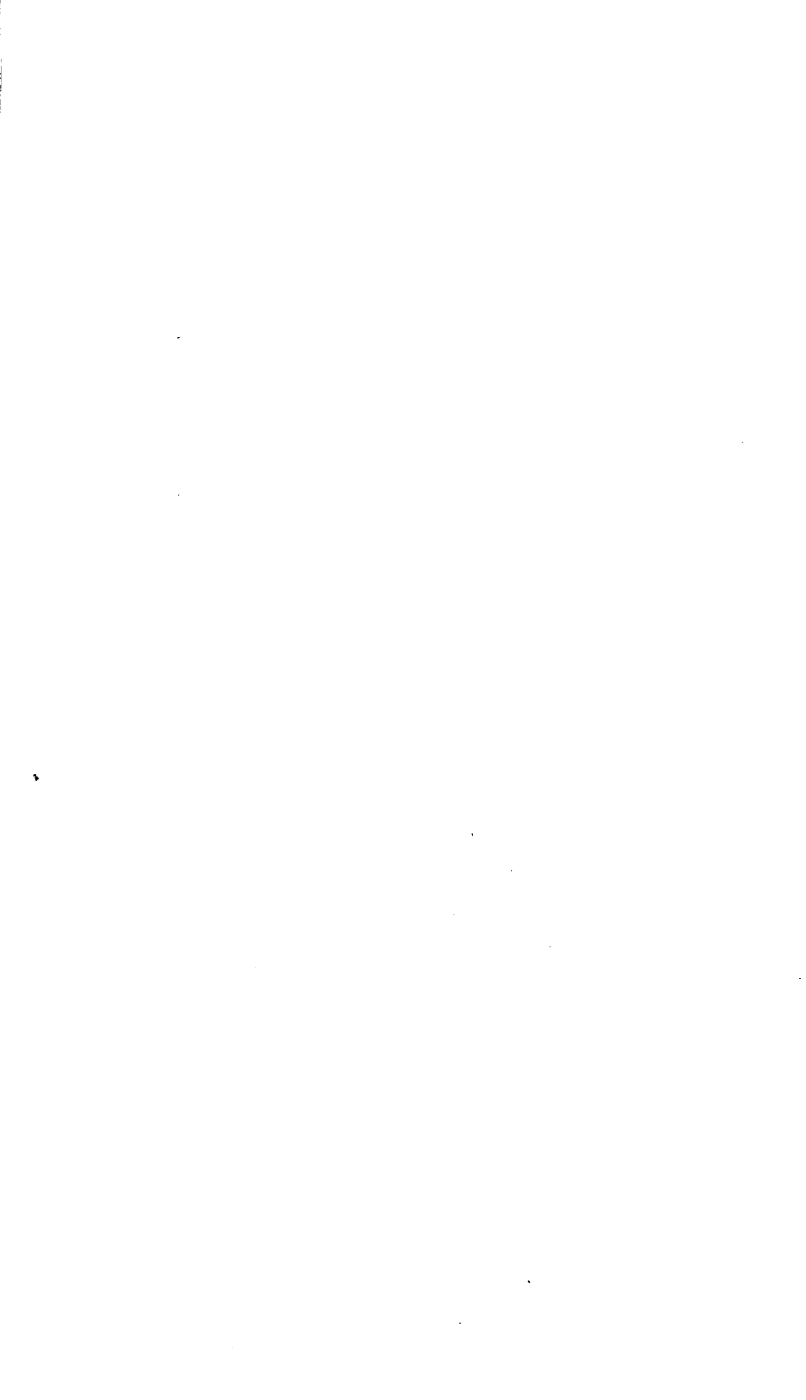
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ERRATA.

- Page 48. line 4. of col. 2. opposite K. insert "C."
- line 24. from bottom, in col. 3. opposite D. insert "20."
- line 5. from bottom, in col. 2. for "117" read "147."
- line 2. from bottom, in col. 3. opposite A. insert "10."
59. line 13. for " $c'b$ " read " $c'b'$."
70. line 8. for "5" read " 5° ."
77. line 1. for "horizontal" read "horizontally."
91. line 2. of note, for "and" read "as."
92. line 7. from bottom, for "one" read "any."
141. line 5. for "64 50 S" read " $64^\circ 50' 3''$."
149. line 12. for "70 16 37" read " $70^\circ 16' 37''$."
- line 7. from bottom, for "31 4 16" read " $31^\circ 4' 16''$."
162. line 1. for "3.1395614" read "4.1395614."
166. line 2. from bottom, for "angles" read "angle."
168. last line of table, for "1848.5" read "1648.5."
188. line 6. from bottom, for "for" read "from."
192. line 14. from bottom, for "2576, in feet" read "2576.14 feet."
- line 2. from bottom, for " $\frac{1}{10}$ " read " $\frac{1}{20}$."
213. in fig. 8. for "a" read "A," and supply "a" at first offset on top of hill.
214. supply "Fig. 9." over the cut, and "a" at the left extremity of its bottom line.





INTRODUCTION.

It does not appear that the ancients were acquainted with the use and advantage of maps up to the time of Anaximander, or about two thousand two hundred years since; and Ptolemy, who flourished in the first century of the Christian era, was the first who used meridians and parallels of latitude. The early maps were chiefly compiled from the itineraries of the Roman and other armies; and we are much indebted to the army and navy of different civilised nations in every period, for the materials from which maps have been constructed.

The art of exhibiting the irregular surface of the earth upon these maps, when their scale will admit of it, is of very modern date, and upon it hinges, in a great degree, the tactics of the modern school.

The general now possesses an immense advantage over the heroes of antiquity, from the facilities of gaining a picture of the country which is the theatre of war, or any part of it that may be necessary upon the spur of the moment: they are drawn by persons appointed for the express purpose, and, indeed, it were endless to insist upon the utility of an art now becoming more generally known and of acknowledged

importance. But there are many others to whom the art, in all its variety, is of equal importance in their several capacities. To civil engineers, geologists, gentlemen of landed property, &c. &c. &c., for conveying information generally of the natural face of a country, these are the advantages of modern over the ancient maps, which, besides their rudeness and inaccuracy, were only lineal, or consisting of roads, rivers, boundaries, &c.

It is curious and interesting to observe how, from the rudest beginning, the moderns have raised the art of representing ground, a word commonly used to express that part of a map or plan which is shaded, so as to give an idea of the hills. The little elevated molehills which anciently, and even a few years since, and still, in some few instances, fill up the spaces between the rivers on maps, have, by degrees, been blended together and formed into regular chains of heights, their magnitude and steepness being estimated by the breadth and intensity of their shade; and the geographer has borrowed from time to time an idea of the military draftsman, until the irregular face of a country is now given in a general manner, more agreeably to its natural aspect than formerly. Hitherto, however, this improvement in maps has been chiefly confined to those intended for military purposes, although of much general advantage.

The establishment at the Tower of London has long existed, and many plans upon these principles have been collected: in some, as old as the beginning of the last century, a great variety of styles, and

traces of an enlarged understanding of the subject, may be seen, according to the merit of the individuals by whom they were drawn : many attempts to imitate nature, as seen from a point above, or according to the orthographical projection, which, considering the low state of water-colour drawing in this country at the time they were drawn, reflect great credit upon their authors. On the Continent great importance has ever been attached to this kind of drawing ; but until within a few years, the Tower establishment and the Royal Military Academy at Woolwich, contained almost exclusively the only persons who, in this country, were qualified for such an undertaking.

The Ordnance Survey of Great Britain, and lately of Ireland, which is perhaps the best ever undertaken, with its adjuncts, under Col. Colby, opened a grand field for the acquirement of topographical knowledge ; and lastly, the Royal Military College has become a school where it has been much cultivated : hence, as might be expected, the British army is now well provided with persons who possess the necessary talent for supplying that army with the most interesting documents of the kind that can be wished for. To make this species of drawing more generally known is the object of the present work ; and it is hoped that the pupil, in every line of life where it can be useful, will find something to interest and instruct him.

When we descend from general maps to plans of particular places upon a larger scale than that upon which maps are generally drawn, they are called

topographical drawings of a country, and are evidently nothing else than a geological representation of its surface, and they are clearly useful to every class of persons who may be interested in their external or internal productions, which may in a great measure be inferred from the very outline they exhibit, still more from their topographical representation: thus minerals will never be sought for but in mountains of a particular structure, nor will agriculture be expected to flourish upon the naked and steep acclivities of such mountains. As far, therefore, as exterior is concerned, a correct representation of the surface will in some respects become an index to the value of the ground. But it has hitherto been the practice to take little or no notice of this circumstance, as, in a military point of view, the height and magnitude of hills and mountains, and not their peculiar conformation, are of the greatest consequence, considering them as positions more or less susceptible of defence. Hence the numerous systems which have been proposed to insure an accurate conception of these peculiar properties of ground. With the engineer, it is true, the conformation should be exactly drawn, or it cannot correspond with the profiles; thus it is more attended to in such cases.

In every branch of human knowledge, when researches and reasoning have laid the foundation of a proper classification, the study of that branch has become easier, and the department of knowledge advanced towards perfection; surely, then, it cannot be improper to avail ourselves of the present state of

geology, I mean the geology of facts, when an art of so much consequence can derive such eminent benefit from it, and in return present the geologist with accurate orthographical drawings of every species of mountain, perfectly intelligible to him, although he has not seen the originals from which they were drawn. Here, then, is the point where this work will be found to differ essentially from most of those at present extant, and here it is proposed to commence the principles upon which it ought to be founded. Geology acquaints us with the fact, that the outlines of the great features of nature, however confused they may appear to a casual observer, are very distinct from each other, so much so that the component parts of mountains are to be inferred from their profile as presented to us upon our approach to them; and an inspection of numerous good topographical plans drawn on the spot, though differing much in style and execution, will confirm this idea. The study of the geology of facts is, then, a primary object with the topographical draftsman, who, by being acquainted with the outline presented by particular kinds of hill, will be better prepared to find peculiar solid figures generated by such outlines, and consequently be more ready in representing them; and this is the principal difficulty in a topographical drawing, because so much depends upon the facility of the individual in producing an exact representation of an object from a different point of view to that from whence it is commonly observed: that part of the operation which is done first, being

only an application of mathematical rules and instruments to obtain figures, either real or imaginary, similar to those which exist, or may be supposed to exist, upon the earth's surface considered as a plane, as it may be within the limits of an ordinary survey.

It is difficult to collect a set of models to exemplify the nature of such a classification as geology presents us: they must be collected from all parts of the world, with great labour and skill, and they will never be so collected but at an immense expence; yet good drawings and profiles will answer nearly as well, and this limits the expence very considerably; but if they are drawn on various principles, some of which cannot give a true representation, then no system like that proposed can ever be formed at all.



PRACTICAL SURVEYING.

BOOK I.

CHAPTER I.

STATEMENT OF THE PROBLEM, AND THE APPLICATION OF THE THEODOLITE TO ITS LINEAR SOLUTION.

Now to facilitate this end, which appears so desirable, we shall suppose the subject altogether new, and consider the thing to be done as a simple problem, thus: — it is required to represent orthographically, upon the plane of the horizon, the irregularities of the Earth's surface, under any given disposition of the light; or, with that light issuing in parallel rays from any place whatever above the horizon.

This, as applied to a model, is not difficult; but in the natural object, it involves many difficulties, which it will be the business of the following pages to remove, for the benefit of the unpractised student.

The smallest hills are too large to be seen at once, nor can we see them, as in the orthographic projection, without we could arrive at a height where

they would be barely visible: we must therefore have recourse to Mathematics and instruments, to enable us to accomplish what is evidently beyond our reach by any other means.

If we consider the level of the sea at high or low water-mark, when the difference is sensible, as the plane from whence we commence our operations (and it will not differ much from a plane in the small superficial area of a few square miles), then it is clear, that, by knowing the perpendicular altitudes above that plane*, and the horizontal distances in any plane parallel to it, points are found in the open space above it, which, when orthographically projected upon the original plane or level of the sea, will exactly define the places of objects situated at those points. To do this is called Surveying. It requires the use of instruments, the most perfect of which is undoubtedly the Theodolite, because it gives the actual horizontal angle without reductions, and accordingly it is used by all scientific surveyors in such operations: but when the obliquity of the planes to which such objects are referred, is not more than a degree or two, and the angular distances are great, then a reflecting instrument will do exceedingly well, although no reductions are applied. We may, therefore, use a reflecting instrument, although not with the last degree of exactness. Hence, before we can represent the solid as required by the problem, we must ascertain its horizontal and vertical dimensions,

* See Levelling.

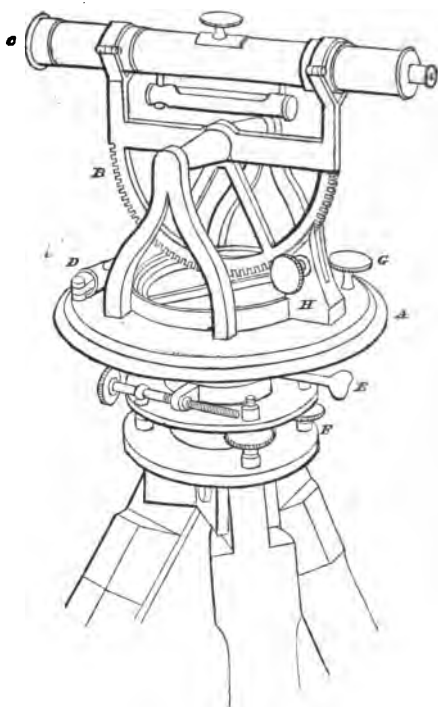
and we are then in a condition to reduce it to a representation which will answer its intended purpose in proportion to its correctness.

Another instrument is also necessary to measure at least one linear dimension, and this is the Chain, or some equivalent for it. A description of the theodolite and chain will now be given.

The theodolite is an instrument used for taking angles in horizontal planes called azimuth angles, or bearings when referred to the meridian, and simply horizontal angles when they only indicate the inclination of one line to another; also vertical angles, or the inclination of any line to the horizon in a vertical plane, as in taking altitudes.

As it seldom happens that three objects lie in a plane parallel to the horizon, and as the triangles they form, when connected by three right lines, would require a reduction to the horizontal plane when taken by an instrument placed in a plane passing through the three points, as in the case of the sextant and other reflecting instruments; it follows, that an instrument which saves the trouble of such reduction, is preferable in most cases, and such is the theodolite. In other cases requiring more expedition and less exactness, reflecting instruments are successfully employed, without regarding the reduction, and, when skilfully applied, are extremely useful.

The manner of using the theodolite, and as much of its construction as may be necessary, will be understood from the following description:—



THEODOLITE.

A strong plate of brass is graduated on its circular edge A ; it is provided with a spirit-level, and sometimes with two ; this has two parallel plates, and four screws underneath, by which it may be placed perfectly horizontal when set upon the legs intended for its support : upon this circular plate is placed a compass to indicate the azimuth, and is generally much used in surveying, although not absolutely necessary.

On each side of the compass are placed two arms of brass, joined at the top, and serving to support an axis which is parallel to the horizontal plate, and which carries a semi-circle B, and also a telescope C, having a spirit-level either above or below it.

Within the telescope is a system of wires, or simply two wires, the intersection of which is in the axis of the telescope, and marks what is called the line of collimation, thus \oplus , or thus \otimes . The latter arrangement is often preferable to the first, and may be easily obtained by turning the telescope on its axis one-eighth of the way round.

To use the theodolite, set it upon its legs by taking two of them in the hands, placing the remaining leg on the ground, and altering first one and then the other, so as to make it stand nearly level, which may be known by the needle and the level upon the horizontal plate at D. Set the verniers upon the horizontal and vertical arcs, both at zero, and loose the clamping screw at E: now turn the whole instrument round until the telescope lies exactly over one pair of the screws belonging to the parallel plates at F, and clamp it gently. In this position, by working the screws of the parallel plates, always unscrewing on one side and screwing on the other, the level upon the horizontal plate may have its bubble placed exactly in the middle; and by a similar proceeding with the remaining pair of screws, the other level on the horizontal plate, if there be one, and also that upon the telescope, may be adjusted — the same adjustment influencing both. But many theodolites,

when small, have but one level on the horizontal limb, and that attached to the telescope answers the purpose of the other, although imperfectly; therefore when this is the case, the clamping screw must be again loosened, and the instrument turned round one quadrant, which brings the horizontal level over or in the direction of the remaining pair of screws, which are then worked, and by this means the horizontal plate is made perfectly level: when this has been carefully performed, the bubbles will rest in the middle of their respective levels in every azimuthal position of the telescope; therefore, we must now turn it gently round until the needle stands nearly N. and S., then clamp it firmly, and by the tangent screw, opposite the clamping screw, the adjustment of the needle to its corresponding points is made perfect.

If we find it does not stand the test of this adjustment, a particular adjustment of its several parts will be necessary, and this must be performed by means of many small screws, not mentioned here: it is a work of considerable difficulty, and must be patiently done at leisure.

As all angular instruments are now provided with a vernier scale, it will be proper here to describe it.

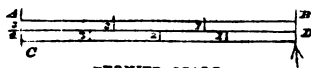
If a circular arc or straight line be divided into any number of equal parts, and it is required to subdivide them again into smaller parts, recourse is had to the vernier scale, the principle of which is as follows:—

Let any circular arc or straight line contain a

given number of equal parts, and another equal arc or line be divided into one more than the given number; then it is plain that the divisions upon the arc or line containing the greatest number of equal parts, will be less than those on the given line or arc, in proportion of the former to the latter; and this is what is called a vernier scale.

Example. Let a given

line A B contain any number of equal parts, and let it be required



to divide each into four smaller parts; take three of the divisions upon A B, and lay them down upon another line C D, which must be divided into four equal parts: now, it is evident that a division upon C D is to one upon A B, in the ratio of 3 to 4, or $= \frac{1}{4}$; and hence, when the scale C D is applied to A B, and moved forward, as each of the divisions comes successively in contact with those upon the fixed scale A B, the dart at D will have moved over $\frac{1}{4}$ of the primary divisions, and the number upon the vernier will indicate how many fourths it has advanced from the last primary division. Again, let it be required to divide the degrees upon any angular instrument into minutes. If the degrees are subdivided into halves, the vernier will be much shortened. Take a circular arc of 29 half degrees, or $14^{\circ} 30'$; divide this into 30 equal parts; each of these will be to the half degrees as 29 to 30, or $= \frac{29}{30} = 29'$; hence, as the dart or zero of the vernier moves from any division, the successive contacts of the vernier

divisions with those on the limb will indicate how many minutes are to be added to the last degree or half degree, to complete the measure of the angle.

For half minutes, it is usual to divide the degrees into three parts, or 20' each; and 39 of these being divided into 40 parts, the latter will each be $\frac{39}{40}$ of the former, and the instrument will read to $\frac{1}{2}$ a minute, or 30"; but in this case the divisions upon the vernier marking the $\frac{1}{2}$ minutes are made shorter than those for the whole minutes, to prevent confusion; and, of course, we have the choice of using them or not: for in many cases the minutes will be sufficient. Some instruments are divided by this contrivance as low as 10" or 15", but the great length of a vernier reading to single seconds has occasioned a very complicated and curious appendage to be applied to large instruments for that purpose.

To take an angle or bearing, set up the instrument as before described; use the milled head G: this will, without derangement, move the telescope until the object is seen by looking over it nearly in the direction of its axis; then with that milled head and another, which moves the vertical arc at H, the telescope must be brought into such a position as for the intersection of the wires to bisect it, either in its height or breadth, according as the altitude or bearing is required; and the number of degrees passed over by the needle, which remains at rest, or what is more correct, by the vernier upon the horizontal circle, is the measure of the angular distance from the magnetic meridian or its bearing, and upon the vertical

arc, the number of degrees passed over by the vernier is that of its elevation.

If the angular distance between two objects, without reference to the meridian, is required, it is evident that, let the zero be wherever it may, the difference of their bearings will be their angular distance; and if this be taken in several positions of the zero with respect to the needle, the theodolite becomes a repeating instrument, and the errors of subdivision are nearly destroyed: this is, however, only necessary on very particular occasions.

It will be easily seen that whatever altitude an object may have, the bearing remains the same; therefore, in all cases where the theodolite is used, and the triangles too small to be influenced by the spheroidal figure of the earth, as in surveys of small extent, it gives a triangle formed by three vertical planes intersecting each other at the angular points, or such as would be projected orthographically upon the plane of the horizon, and, consequently, that no reduction is necessary; but when we actually measure up or down a slope of great steepness, then, indeed, the line must be shortened in proportion of radius to the cosine of the angle of elevation above the horizontal plane; but it is only in surveys of particular places, laid down on a very large scale, that this difference is sensible, or in long lines measured over a series of very uneven ground. In surveys of large extent, instruments of a more complicated construction and larger size are made use of for obtaining the greater triangles, in which corrections are necessary,

because the radiation of the perpendiculars passing through the angular points towards the earth's centre become sensible; and many other niceties must be attended to, which, in triangles of only three or four miles in extent, are altogether insensible, and therefore in common surveying are neglected; nor would the small theodolites employed in such surveys show them, as is done by the larger ones constructed on purpose.

Large theodolites are not necessary on small scales, nor indeed for any other than nice purposes; and therefore the author contrived one about 3 inches in diameter, many years since, made by Troughton and Simms, which has been found to answer extremely well. It differs from all others in having the supports for the telescope extremely small and light, being inclosed in a cylinder 4 inches high, with the necessary perforations to admit light, &c. There being no room for a compass inside, it is placed on the top in open daylight, which is very convenient; and it has a card instead of a plain needle: this sooner ceases to vibrate; and on common occasions, serves to place it very nearly level, without having recourse to the repelling screws underneath. Another level should be placed on the horizontal plate, at right angles to that on the telescope. The instrument weighs, with the case, 2 lbs. avoirdupois; and a brass cover is made to screw on over it, at a distance sufficient to protect the whole when not in use, weighing $9\frac{1}{2}$ oz. avoirdupois. If a fall or blow happens, the cover is deranged, but the inside is safe; whereas, in the usual

instruments, the whole is sometimes forced off the plate, and it never can be exactly adjusted afterwards. With silver limbs divided to minutes, and a lens to read both horizontal and vertical arcs, it costs 11*l.* 11*s.*

CHAP. II.

SURVEYING BY THE THEODOLITE.

SURVEYING is, then, the art of obtaining the dimensions and forms of all figures, however irregular, upon the earth's surface, whether the boundaries consist of roads, fences, margins of lakes, coasts, rivers, &c. &c. &c. such as they would be orthographically projected on the plane of the horizon.

As every plane figure may have a right-lined figure either inscribed within, or circumscribed about it; as the irregularity of its boundary can always be obtained by perpendicular ordinates; and as similar right-lined figures have their angles alike and their like sides proportional,—it is evident that if a sufficient number of the sides and angles of any figure are measured on the ground, a similar figure may be projected upon paper, and by any scale that may be required. The actual performance of these operations constitutes the practice of surveying and plotting, the operation out of doors being called by the former, and laying down the same on paper by the latter name.

To take the most common case; if lines are measured connecting two of any three objects, and the angles contained between these two lines, or

between each of them and the meridian is also measured, that triangle can be projected without measuring the third side; a series of distances and angles thus measured through a road, by the margin of a river, or any boundary whatever, are all that is commonly necessary for laying down a plan of the same; but if its irregularity is such that it does not coincide with the straight lines measured, then perpendicular ordinates, called offsets, are also measured on one or both sides, where necessary, and the exact form is thereby obtained.

The chain employed in common surveys is 66 feet in length, or four poles of $16\frac{1}{2}$ feet each: it is made of iron or steel wire, and consists of long and short links alternately, these short links or rather rings being placed between the eyes of every pair of long ones, the handles at each end, with the adjoining half-links, count as links; this gives 100 divisions, and they are called 100 links; they are numbered at each 10 from the ends towards the centre by pieces of brass divided into leaves, but the centre brass or 50 is circular.

The angles made by the measured lines with the meridians having been mentioned, it is necessary to explain why this method is preferred in practice, and it is shortly this:—when the lines are laid down upon paper, many of them are so short as to produce *certain* errors in adjusting the protractor to them for laying down the next angle—these errors would therefore accumulate and produce much confusion; but if a number of lines are drawn across the paper

exactly parallel to each other, and the protractor constantly adjusted to one of them or an intermediate parallel, then the errors are considerably diminished, or altogether disappear, because every bearing is referred to a parallel or meridian with only its own error, instead of being referred to a line already laid down, which, if short, would certainly *be liable to error*, and thus an accumulation would take place; besides, this method has many advantages which practice alone can show, in respect of expedition, convenience, and so forth; it is, therefore, generally followed by the best surveyors on all occasions.

Where the ground is not very irregular, a survey is thus conducted; and we shall afterwards show how the same must be done when that is the case.

We shall suppose that a series of triangles have been accurately formed by large instruments, and that at least two of these fixed points are contained within the limits of the survey. We must then set up the instrument at one of these places, and carefully ascertain the magnetic bearing of the other; this may be done at any convenient time while prosecuting the work. Now these two points being laid down upon paper by their distance, and connected by a line, we lay down one magnetic meridian by it, the rest are ruled parallel to it, and the business of plotting is then commenced.

In surveys of estates it may be sometimes necessary, from their great extent, to form two or more large triangles from a measured base as a check upon the work; but we shall first show the general method

of surveying, which in that particular application merely requires greater nicety, and that more offsets should be taken.

We begin by setting up the theodolite as before directed. A stick with a bit of white paper on its top, or a pole with a flag upon it, is carried forward, and placed on a convenient spot for the instrument to stand, as well as for seeing forward to the next station, a name given to every place where an angle is taken; the bearing of the forward station is to be taken to the bottom of the stick or staff, and to be read off and registered in the field-book. If any house, tree, or remarkable object presents itself on either side of the station lines, not more than one mile from it, the bearing of such objects should also be taken and entered in the field-book. This is to be done from at least three different places, as they are good checks, and in small surveys will supply the want of triangular points; and, besides, it is the way to determine their true situation on the future plan. These intersections may be more frequently made with advantage if the objects are not too distant; for when the lines drawn to them are nearly two feet long upon the plot, it is difficult to protract them with sufficient accuracy — but when shorter they are an excellent proof of the work.

The measuring is thus conducted. Place the chain exactly straight between yourself and the forward station; let the leader who draws the chain pull it tight, and place an arrow or iron wire pin exactly at the end of it; then both walk forward, the follower

noting in his book, which will hereafter be described, the perpendicular distance to any bend in the hedge, road-side, &c., as well as its direct distance from the last station, sketching in whatever may be necessary: it is best to take offsets on each side before you stir from the first station, and if you see others will be necessary, not to let the leader move until you come up to him, otherwise you will not see their distance from the first station. When you arrive at the forward arrow, the chain must be again laid straight, and being kept so until all the offsets are entered, another arrow must be placed at the end of it by the leader, the follower always hanging upon the fingers of his left hand those arrows he may have taken up, and never doing so until he is ready to go forward, lest he should lose his place, and be obliged to measure his line over again: the leader may prevent this inconvenience by scratching a cross on the ground before he sticks in the arrow. This must be repeated through the whole line, and the number of arrows at any time upon the follower's fingers will show how many chains he has measured, while the tens and units are counted on the chain as it lies upon the ground: the chains are always entered as hundreds of links, to avoid the use of decimals.

As great mistakes may happen in conducting this part of the work, too much care cannot be taken, especially when after ten chains have been measured the follower has to give the leader the ten arrows; the best way is for the leader to wait at the end of the eleventh chain until the follower comes to him,

and to put down one arrow immediately after he has received them, the follower having previously marked 1,000, 2,000, &c. when he took up the tenth, twentieth, &c. arrow. A stick or pole being left at the last station, and having arrived at the next forward station, the instrument must be placed over the point measured to, and the same operation repeated as at first. To insure the accuracy of the bearings, the instrument should be turned round to zero, after they have been all taken and entered; and if it has not been deranged in taking them, the needle will stand right, or *nearly so*: if it is out several degrees, an error has been committed, and must be immediately rectified, by going back to the last station, or by taking the bearing back again.

We have said *nearly so*, for there is a variation of the needle by which it will differ at different periods of the day and year; the maximum of these diurnal variations takes place between noon and three o'clock in the afternoon, and the greatest diurnal variations generally take place in April, May, June, and July; the greatest observed in London, in 1759, were 13' 21" in June; and the least 6' 58" in December. It may be proper to mention here, that by late discoveries well authenticated, it has been found that all perpendicular objects whatever, as houses, trees, &c. &c., have (at least in north latitudes) a north pole at bottom and a south pole at top, and that they exert a sensible influence upon the needle: it may, therefore, when delicate instruments are employed, be worth while to avoid placing them near any verti-

cal object if possible; at any rate a blacksmith's shop, and every place where much iron is deposited, must always be avoided.* A multitude of minor irregularities, chiefly the result of late discoveries, are now known to exist in the magnetic needle; fortunately, however, many of them are too small to materially affect those works where it is employed.

If this should weaken our confidence in the needle, surveying may be conducted without absolutely depending upon it, after having used it at first to begin with. This is called surveying by the *back angle*. It appears from numerous observations, that the diurnal variation commences two or three hours before noon, having previously returned to the position it had on the preceding day, and been stationary during the night: if, therefore, we begin early in the day, it may be depended upon then, and need not be trusted to afterwards, except as a check against great errors in the angles: to do this, we must always set the instrument to the last forward angle, and adjust it to the back station, then screwing it fast by the clamping screw, the new angles may be taken without reference to the needle, otherwise than as a check against an error which sometimes happens, by taking the fives for tens in the degrees upon the limb, or by making a wrong figure in the field-book, in either of which cases a return to the last station is necessary, or to use the needle as at first.

We have mentioned these things, because it ap-

* An error of 2° or more has often been detected in this vicinity, from local causes which cannot be well accounted for.

peared that they ought to be known to the young student; but we must add, that the greatest error from the diurnal variation would only be 31,067 links, and the least 16,212 links in a distance of one mile, which, if the survey is laid down upon a scale of four inches to the mile, will be equal to 0.0155 inches and 0.008 inches respectively, quantities not greater than the diameter of a hole made by a bad pair of compasses, and in the usual distances of a few chains they are insensible; but for particular plans upon a large scale it is of consequence, and therefore they are generally surveyed by the *back angle* — and hence a rule in these cases not to trust to the needle, except when unavoidable.

The preceding facts will show decidedly how necessary large and accurately divided instruments are, when used for very distant objects, as in trigonometrical surveying; and also *why the needle cannot be employed upon such services.*

It is usual at the conclusion of every day's work, especially when surveying by the *back angle*, where it is *indispensable*, to take what is called a *return angle* at the last station: this is, in fact, taking the bearing of any well-defined object, the more distant the better, and noting it in the field-book; and also to mark the point of ground over which the instrument stood, by cutting a deep cross, driving a picquet, or otherwise. The use of this is, that when the work is resumed, the instrument may be adjusted to that distant object as it is to a back station, and thus the accuracy of the next forward angle is secured;

or, in other words, the identity of the situation of the instrument as it last stood in that place; and it effectually destroys the influence of the diurnal variation, as it does not matter at what period of the day or year the work is resumed.

The instrument should in strictness be brought exactly over each station point; to save time, surveyors are accustomed to place it at once so nearly over this point and also horizontal, that a small stone dropped from the centre, underneath the instrument, will fall exactly, or very nearly so, upon the station point, and but little levelling is necessary: this precision is soon acquired by practice. An error of half an inch on either side of the station might produce an error of near 2'' in the angle, when the forward station is distant one mile, but the theodolites in common use will measure no nearer than one minute; the exactness before mentioned is therefore sufficient in common surveying.

In surveying for content, an offset-staff of ten links long, divided to links, is used for measuring the perpendicular ordinates; but on the four-inch scale, or even six or eight inches or more, the greatest error that can be committed by pacing them can hardly exceed one or two paces in twenty, even by different persons, and this upon the 8 inches scale would amount to no more than 0.004 inches and 0.008 inches, quantities too small to be discerned without a microscope, and not affecting the *general* accuracy of the work, nor its content sensibly. Upon a 20 inches scale a like error would be 0.01 inch,

and 0.02 inches upon one or two paces respectively, which is but the breadth of a pencil line.

If triangles have been previously laid down by trigonometry, we obtain what are called trigonometrical points: whenever our survey leads us near any of these, it is always proper to survey up to them if accessible, and if not to frequently intersect them, for they are carefully laid down in order to insure correctness in the work. It is best always to survey the boundary of a piece of work first, whether it contains points or not, and then to fill in the intermediate roads, &c., commencing with the longest. These trigonometrical points are absolutely necessary before a work of large extent is commenced, on any scale.

As there are several errors to which a survey is liable, it remains to point out the causes of the chief of them, as it will show how much care is necessary in every part of the work.

1st. Those belonging to the instrument itself. It may be badly centred, and equal arcs will not be cut off on opposite parts of the limb; this is seldom considerable. It may be badly divided; this is not often the case in the improved method of dividing instruments, but shows itself when the vernier does not embrace equal arcs on every part of the limb, and so does the bad centring also. Both these sources of error are remedied by taking the mean of several angles on different parts of the circumference; this is, however, only necessary where great accuracy is required, as in fixing minor points. The

needle pivot may be bent or badly formed: in this case, the needle will not divide the circle around it equally, and then only the north end should be used, and that always set to the north point of the compass box; then its error, abstracting from the effects of diurnal variation, is a constant quantity, which may be found and allowed for in laying down the north point on the plan. The wires may not cross each other in the axis of the telescope; they must, therefore, be adjusted. The bubbles of the levels may not always remain in the centre when the instrument is turned round; they, therefore, require adjustment. An error may exist in the vertical arc when its level is right; this may be found by reversing the telescope, and becomes a constant quantity to be added or subtracted in all elevations or depressions.

2d. The errors in the protractor may be those of bad division or bad centring, as in the theodolite, and may be corrected nearly in the same manner; but occasion much trouble.

3d. The errors in laying down the work may proceed from expansion or contraction of the paper, and are then to be avoided by the application of heat or damp, or still better by first mounting the paper on a board. If from want of coincidence of the protractor with the ruled meridian, or those not being parallel, or too large holes made with the compasses, more care is necessary.

4th. The scales may be ill divided; a surveyor can make them for himself, and if upon the same

paper as the plot, they will alter with it, and be always right. In short, these numerous sources of error are enumerated more to guard the pupil against them, and teach him where to suspect, in case of inaccurate workmanship, than from the extent to which they exist, as far as the instruments are concerned, most of them being *small*, and when good instruments are employed skilfully, not often giving much trouble. It may be necessary to remark, that the chain should be measured from time to time, to see whether it keeps the same length, and if not, the difference should be noted and allowed for.

The best way is to measure 66 feet on the ground, and drive a peg at each end; then if the chain has lengthened, as it most commonly does, it will be immediately discovered.

The fitness of a theodolite for the purposes to which it is applied, has occasioned the best artists of this country to turn their attention to its improvement, and the splendid instruments they have produced have been applied successfully to the mensuration of degrees of the meridian and of longitude, by which means it was hoped the real figure of the earth might be at length determined. In the present instance of surveys, not larger than an English county, we have no occasion whatever to depart from our original supposition of a plane surface to the extent supposed, for a degree of 69.1 English miles, considered as a curve, measures about 6 feet longer than its chord; we cannot expect our series of linear

measurements, with all possible care, to come nearer the truth than that when extended to several miles.

We have now laid down the method of operating upon ground level or nearly so. We must next consider the case of a mountainous district, where the acclivities are considerable, and we shall defer the account of the field-book, and description of the protractor, until we come to treat of the plotting as a separate article. If the slopes were inclined planes, since the theodolite would give us their angles of inclination to the horizon, we could easily find the space they should occupy upon the horizontal plane, but they are commonly curved, so that it is plain the reductions would be innumerable, and, consequently, the process too long for the time which could be allotted to it; if, therefore, we select some place where a base can be measured either on a level or a gentle slope, — if the latter, it can be reduced to the horizontal distance, — and if the angle be taken between signals in proper situations from both ends of the base, and from one to another, a series of horizontal triangles will be formed, and the reduced distances or true orthographical projection will be found by simply laying them down on paper by their calculated sides, or by the angles on any given scale; but the calculated distances are most to be relied upon, and even then involve considerable uncertainty in all cases of acute intersections. This being done, the intervening mountains and hills must be sketched by the eye, as will be shown hereafter. In every

case of roads, boundaries, &c., which cannot be obtained by such means, we must have recourse to reducing the measured distances as we go on, according to the following scale, having surveyed them in the manner before described. By this method a survey is accurately carried on, and by way of confirmation the distance between two convenient stations may be afterwards actually measured, which most effectually corroborates or disproves the preceding triangulation. It is a good way in all triangulations to work round to a former point, and thus calculating or laying down a triangle by new data, one side of which was formerly laid down or calculated, we shall see if they agree or not.

Two distances obtained by the sextant from different data are as follows : —

15915.85	}	feet respectively,
15916.		
7393.1	}	feet respectively,
7396.8		
4079.8	}	feet respectively,
4078.6		

with many others equally satisfactory.

TABLE OF REDUCTIONS TO HORIZONTAL BASES,
IN LINKS AND DECIMALS.

Degs.	Links.	Degs.	Links.
5	00.4	26	10.1
6	00.6	27	10.9
7	00.7	28	11.7
8	01.0	29	12.5
9	01.2	30	13.4
10	01.5	31	14.3
11	01.8	32	15.2
12	02.2	33	16.1
13	02.6	34	17.1
14	03.0	35	18.1
15	03.4	36	19.1
16	03.9	37	20.1
17	04.4	38	21.2
18	04.9	39	22.3
19	05.4	40	23.4
20	06.0	41	24.5
21	06.6	42	25.7
22	07.3	43	26.9
23	07.9	44	28.1
24	08.6	45	29.3
25	09.4		

The foregoing table is formed by simply subtracting the natural cosines from radius; the intermediate numbers may be found sufficiently exact by simple proportion: by removing the decimal point two figures to the left, they will become the number of 1000ths, and this to be subtracted from any distance whatever. Rule. — As 100 links to any number of links measured up or down a slope, so is the tabular number to the reduction required; therefore the distance in links multiplied by the tabular number,

and two figures cut off from the right hand, or three when decimals of links are used, will be the answer: —

Example. — Angle of elev. $29^{\circ} 33'$, dist. measured 600.

	Links.	
Angle, $29^{\circ} 33'$. . .	12.9
× by		600
		77.400

and $600 - 77.4 = 522.6$, the }
reduced base. }

At 15° , which is a very tiring ascent for men and animals, the reduction is .034 of the whole distance or 3.4 links in each chain, at 6 chains this amounts to 20.4 links; upon the scale of 8 inches to one mile this would be about .02 of an inch, a quantity nearly insensible; upon a scale of 20 inches to one mile, or 4 chains in one inch, it would be a little more than $\frac{2}{40}$ of an inch, which is a very sensible magnitude. For the reasons before given it is better to avoid these reductions as much as possible by the method of intersections, for the slope sometimes changes so much at short distances that the work would be thereby exceedingly prolonged. Whenever it is done, a staff of the same height as the instrument should be set up forward, and after a careful rectification by its levels, an angle of elevation or depression should be taken to the top of the staff, and noted in the field-book between the proper limits of the chain-line.

Since, as we have seen, the difference between inclined lines and their horizontal bases can only be shown on large scales, or when the inclined lines make large angles with the horizon, it is the practice with most surveyors to disregard them altogether, wherever the nature of *the ground* will admit of it,

and to compensate for this by a certain tact which practice gives them in a very eminent degree. The experienced surveyor well knows that it is next to impossible for him to measure so many lines of different lengths and over surfaces of different degrees of smoothness with mathematical exactness, and his experience teaches him also that, from a combination of circumstances, his series of bearings commonly run too long, he therefore shortens each line a little whenever they are not measured over level ground; but when the slopes are very steep, and particularly if also long, either horizontal intersections or actual reduction must be resorted to.

In surveying countries not having considerable elevations, we may safely trust to the preceding method, and if hills should arise suddenly, as they sometimes do, which give us reason to expect our chain lines will run too long, we may avoid this inconvenience by carrying our measurements around these hills as much as possible, by which means we reduce the angle of inclination, and intersections will determine the points situated upon the hills correctly. In fact, we should avoid these reductions as much as possible, for they consume much time, and also, as we have before observed, because the hills are usually curved, which would make innumerable short stations necessary; whereas to insure accuracy, the measured lines should always be the longest possible, and the angular bearings the fewest. This is laid down as a principle in surveying, both to save time and to lessen the sources of error.

CHAP. III.

FIELD-BOOK. — USE OF PROTRACTOR. — PLOTTING AND
CALCULATION OF CONTENT.

WE must now describe the Field-Book, and also another instrument called a Protractor, which is necessary, in order to lay down upon paper the angles which have been measured in the field.

The field-book has been mentioned under the article SURVEYING; but it must now be more particularly detailed. It is used to enter the bearings and distances taken with the theodolite and chain. It is very essential that this register should be kept as neatly as the circumstances of its being held in the hand while writing, and also while holding the chain, will allow: with experienced persons these impediments are entirely removed by habit.

As every one has his own method of keeping this register, we shall only point out that which we prefer and always use from its simplicity, which in this is everything. It should be free from ambiguity, so as to allow of any other person plotting from it in the event of the work not being finished by the person who began it.

Two parallel lines are ruled through the middle of each page, about an inch apart, to admit of writing numbers of four or five figures: this is all the preparation necessary.

The first station is entered at the bottom of a page by a small circle, with a point within it, thus ○.

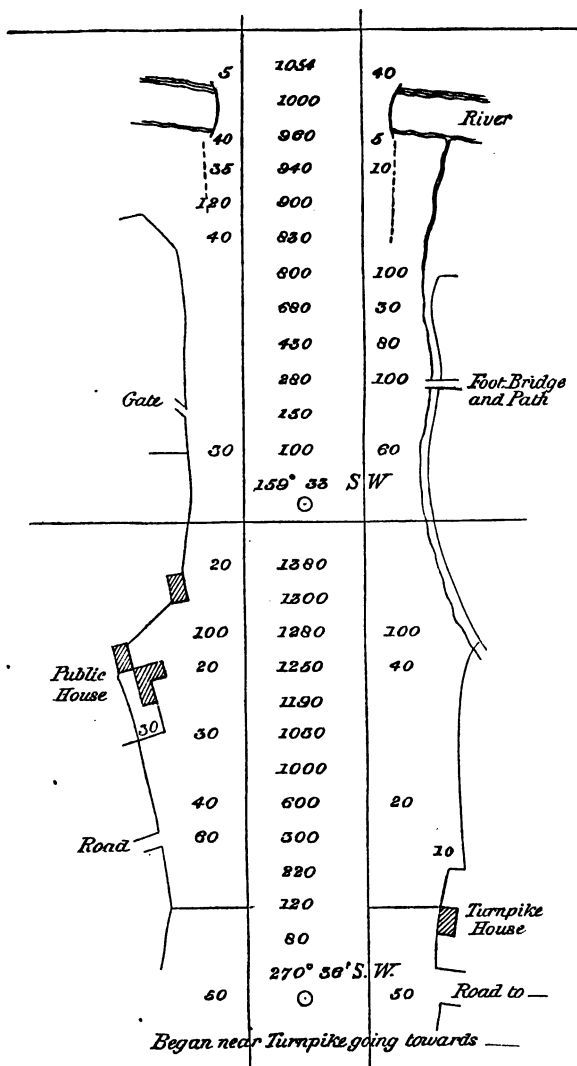
The intersection to distant objects, if any, should always be entered first, and the forward bearing afterwards, to prevent mistakes in plotting by laying down the distance upon a wrong bearing; then the distances at which offsets have been taken, also those offsets themselves on the right or left as the case may be, a line being drawn by either or both sides, following the principal bends of the fences, &c., with a sketch of the houses, &c. &c. &c.; and lastly, at the end of each measured line, the total distance from the last station is entered with an offset on one or both sides, and a line is drawn across the book.

A new station is then marked, and the same process repeated until the day's work ends, at which period a double line is drawn across the book.

The annexed specimen, from an actual survey, will make the following instructions more clearly understood.

If any wrong figure is entered, it should be corrected as soon as discovered, not by an attempt to improve or alter it, which is losing time, and occasions uncertainty, but by running the pen through it, and entering the correct number at the side of the book exactly opposite the false one.

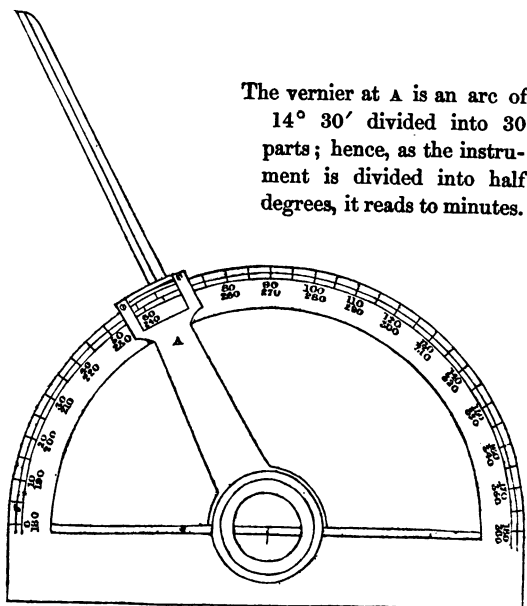
The reason why the field-book is begun at the bottom of each page and written upwards, is because the space contained between the two parallel lines represents the chain line, and as we are always measuring forward or from the last station, it is most



SPECIMEN OF FIELD-BOOK FROM AN ACTUAL SURVEY.

natural to enter the numbers, &c. in that direction ; whereas, in the usual way of entering anything from the top towards the bottom, the work would be inverted ; besides, in plotting from a book, the method prescribed has considerable advantages.

To make this part of the subject still more intelligible, we shall, when the plotting is described, give an entire field-book, and the small plan belonging to it.



THE PROTRACTOR.

The protractor is an instrument which performs the

same office upon the paper that the theodolite does upon the ground: it retraces our steps, and enables us to lay down exactly the route taken by the other instrument.

Protractors are made circular or semicircular; the latter is most convenient, and will be mentioned first. A semicircle of brass is connected with a flat brass ruler, the fiducial edge of which is a diameter of the divided semicircle, or rather it is formed of one solid piece (see opposite figure); and it has generally upon it a diagonal scale of two and four inches to a mile. Upon a centre of a peculiar construction, an index, carrying a vernier, is made to move freely; it has a long projecting end, which, if the instrument is truly made, points exactly to the centre of motion, and cuts off the required arcs upon the limb; this limb is divided into 180° , and numbered so as to read to 360° , that it may correspond with those theodolites which are so numbered, the vernier commonly dividing the degrees into minutes. Some are divided to 180° both ways. As the zero of the theodolite is always opposite the north or south points in surveying, and the protractor numbered so that those numbers above 180° regularly correspond with those below it, and distant from them one semicircle, it is indifferent upon which half of the limb in the theodolite, or which set of numbers on the protractor, the angle is read: thus $187^\circ 15'$ is the same as $7^\circ 15'$; also $269^\circ 4'$ is the same as $89^\circ 4'$, and so forth. This particular arrangement of the numbers, and that of the use of the instrument we are about to describe,

renders all addition or subtraction unnecessary, and thereby much facilitates the process.

The zero is always upon a meridian; the angle is always, when surveying by the back angle, read indifferently on either half of the instrument between 0 and 180, or between 180 and 360: now all ambiguity disappears from simply entering in the field-book the direction of the bearing, that is, towards what point of the compass it is taken. Thus $187^{\circ} 15'$ and $7^{\circ} 15'$ both give the same line upon paper, but it must be noted whether we looked towards the north-eastward, or towards the south-westward; also $269^{\circ} 4'$ and $89^{\circ} 4'$ are each inclined to the meridian in an angle of $89^{\circ} 4'$, but they may be either S.W. or N.E. These things being well understood; the method of laying down the angles or plotting them is simply thus:—

A large sheet of paper is carefully covered with lines correctly parallel to each other, about an inch and a half apart, ruled through its whole length,—these are magnetic meridians: one end of the centre line is marked north, and the other south; the east and west points are also marked upon the sides of the sheet right and left of the meridian.

Now a place being chosen upon the paper for the first station, it is marked with the point of a hard pencil, and a small circle drawn around it; the protractor is then set to the angle in the field-book, which shows the first bearing, and adjusted to that meridian which is most convenient for ruling the bearing from the station in its proper direction. To

prevent it from slipping, a ruler is placed next the person who uses it, and behind the protractor; this keeps it steady, and allows of it being adjusted to the station-point more exactly, by sliding it until the projecting fiducial edge passes through the station.

A further advantage is gained by using parallel rulers, for besides keeping the protractor steady, if no meridian is conveniently placed to lay down the required bearing, this instrument will supply the place of one without actually ruling it wherever it may be wanted. There are rulers made for this express purpose, one half of which can be screwed fast upon the table, and they are very convenient when much work is to be laid down, particularly if upon a small scale. When the projecting part of the moveable index is made to pass through the station, a fine pencil line is drawn by it; this is the required bearing, and must have the distance from it to the next station laid down upon it from any scale by the compasses: a small circle is drawn around this new station, and the same process is repeated with great care through all the stations. If the last bearing and distance have been measured up to the first or any other station, and they are plotted right, it is obvious that the last bearing will pass through that station and the distance fall upon it, if the measurement is also right; this is called closing correctly.

The distances should be very carefully taken from the scale, which should be a good one; and we have already remarked that there is a natural tendency, from the little irregularities of the ground, which it

would be endless to reduce to their horizontal lines, and also the holes made by the compasses for all series of distances, to become a little too long.

This method of plotting is so expeditious, that we have seen fifty bearings and distances laid down in one hour, when the scale was about two inches to a mile: this rapidity is, however, not very common.

Now the pupil will see more clearly the reason for entering the forward bearing last, and the intersections before it; for if any intersection is entered last, he is apt to plot that instead of the required bearing.

It is therefore a rule, after laying down all the bearings, and thus ascertaining the correctness of our work, to plot all the intersections by themselves, rejecting those that may not be of material consequence, as we sometimes find more than we want.

We have supposed the instruments free from the error of subdivision and centring, and they generally are very nearly so, or they would consume too much time in applying the corrections; they should, therefore, be obtained from the best makers, and carefully proved before they are used.

The most common error likely to affect the accuracy of this part of the work is, that arising from the fiducial edge which passes through the centre not being parallel to the back of the instrument where it rests against the ruler; but in this case the instrument may always be adjusted to the meridians by the interior edge, and not indifferently by either edge, the rule being employed merely to keep it

steady, or assist in transferring it to another place; or if the back of the protractor is employed for that purpose, and the ruler itself adjusted constantly to the meridians, then the constant error must be found and allowed for, in joining on to any work laid down by another instrument, as also in laying down the true north point.

Every bearing and distance must be marked in the field-book with a pencil as soon as it is plotted, or there is a chance of mistakes, by doing them twice over, or leaving some out.

When this part of the plot is finished and found to be correct, we may either plot the offsets upon that, or *dot* the stations down on a clean sheet of paper, and do them there; in either case we proceed as follows:—

Take the distances upon every chain line with a pair of compasses, and lay them down with the corresponding offsets, which being connected by pencil lines form the irregular boundaries of roads, &c.; or still more expeditiously, by using the edge of a plotting scale, or a piece of paper having the proper scale upon it, both for the distances and the offsets; the houses must also be put in at this time, as their situation, when not depending upon intersections, is always obtained by offsets. It is usual on scales not greater than six inches to one mile, for an experienced person to judge all the shorter offsets, as he soon becomes accustomed to the scale, and it saves a vast deal of time; but upon large scales they must be laid down by measurement.

When it is of consequence to preserve the stations and lines, as in the event of laying down more work, they are drawn in red; and if, instead of finishing upon the rough plot, we do so upon a clean sheet of paper, the station lines are all rubbed out as soon as we have passed over all the roads, boundaries, &c. with Indian ink; the houses are to be made red, and the rivers and streams blue, but no red can be used safely before the Indian ink shading of hills, &c. is finished, because it would wash up; we therefore leave them in pencil until the last. We are now ready for sketching in the hills; for this purpose trace upon thin paper all that has been before laid down, in small portions, and taking them on the ground, draw it in upon the spot: it is obvious, that the more objects intersected, and the more numerous the roads, &c., the easier will be this part of the operation, which, in a deficiency of these data, must be done by pacing, or other means more easily understood, by seeing the actual performance of them, than by a circumlocutory explanation; yet this will be attempted in the following part of the work.

The sketching being finished, it is transferred to the original plan, and the hills drawn with Indian ink or in colours, as may be thought best; but in the latter case grey should be substituted for Indian ink in all those lines before directed to be drawn in the latter colour. We now also put in the woods, commons, and every other object upon the surface, and it remains to lay down the true north point upon our plan.

This may be done by finding the variation of the needle, by equal altitudes of the fixed stars, or by an easy calculation in spherical trigonometry, having the altitudes of one or more stars taken on each side of the meridian for greater accuracy.

In doing this, an instrument in good order must be used, and particularly the needle-pivot must not be excentric, otherwise a correction is necessary from that circumstance. To obviate that and other inconveniences, we may use a method wherein the needle has no influence whatever, although the variation of that particular needle may be inferred from it. This will be mentioned in the book devoted to military sketching. If the variation is well known where the survey is made, and the needle we use is tolerably correct, these latter proceedings are, of course, unnecessary. A scale must also be added in some convenient place; this may be dotted down from the rough plot, upon which it is better to have made one, than to use any other scale whatever: for if the paper alters, as it will very much in great changes of weather, from damp to dry, the scale made upon it will always fit the work better than any other.

Much time may be saved by dividing the rough plot into small portions, and sketching upon them; but this must not be done until it has been transferred to the clean paper for the plan.

There is also a Circular Protractor, which is thus used: a single meridian is ruled near the centre of

the paper, with an east and west line across its centre; the protractor is laid down upon it, having the zero, and the 180th degree exactly over the meridian, and, consequently, the 90th and 270th degrees upon the east and west lines: the centre of the instrument will then be correctly placed over the intersection of these two lines, where it is held fast by four points, which pass through different parts of its circumference. The cardinal points are marked as when the semicircular protractor is used.

The stations are numbered in the field-book; and without the protractor being lifted off the paper, it is set to each angle, and a short line drawn by the projecting fiducial edge of the movable index, which line is numbered to correspond with the angle in the book: when as many as may be thought necessary are thus plotted, the instrument is removed, a parallel ruler is applied to each of the short lines and the centre, and each bearing transferred to that part of the paper where it is required, the distances and offsets being laid down from the book, as before described.

Some protractors are furnished with a point, and not a fiducial edge, and the dots made by it serve instead of the short lines: some have two verniers and two points.

The advantages attributed to this instrument principally arise from the two verniers: when it has them, they enable us to take two readings, it is true, and may serve to correct an angle; but it is not,

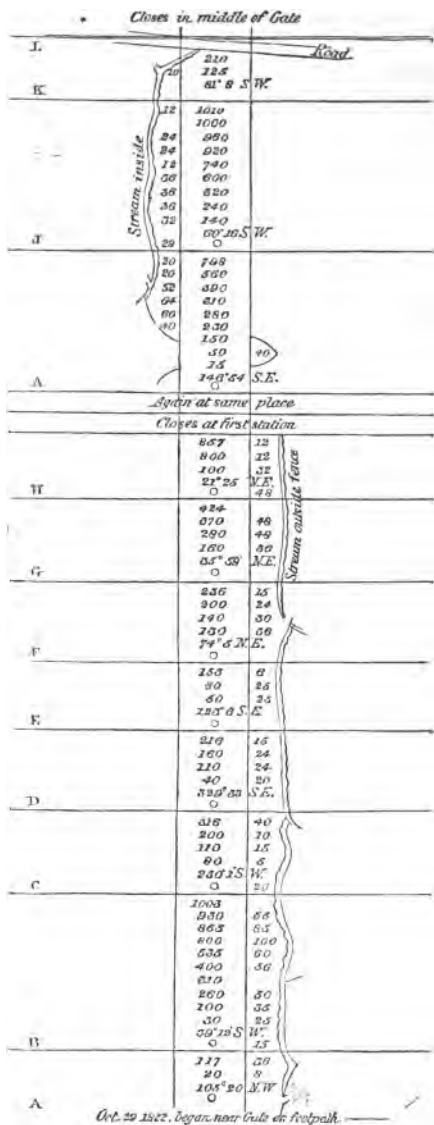
therefore, more proper for laying down trigonometrical points or triangles, which require great accuracy ; for as the distances between such points ought always to be calculated, they are much better laid down by beam compasses, thus avoiding all errors of division in the protractor, and some others which would arise from the use of angular instruments : besides, it must be remarked that the semicircular protractor, although furnished with only one vernier, can be used exactly in the same manner as the circular one, and with nearly the same advantage. The inconveniences of the circular protractor are these, viz. that many of the bearings very frequently come so nearly together as to create much confusion, and render mistakes probable, and, indeed, very likely to happen, unless several places are chosen for fixing it at different times ; also, the transfer of the bearings is attended with great probability of error, when they happen, as is often the case, to lie at a great distance from the place where the protractor has been used.

We shall now present the reader with a real field-book, and the plan which was plotted from it. (See the following page, and *Plate I. fig. 2.*)

For the convenience of reference to the Plate, the stations are marked A, B, C, &c., to correspond ; but this is not absolutely necessary in practice.

The dotted lines represent the triangles and trapezia the fields have been formed into, by taking in and throwing out parts of their borders — a practice

FIELD-BOOK OF SURVEY LAID DOWN IN PLATE I. FIG. 2.



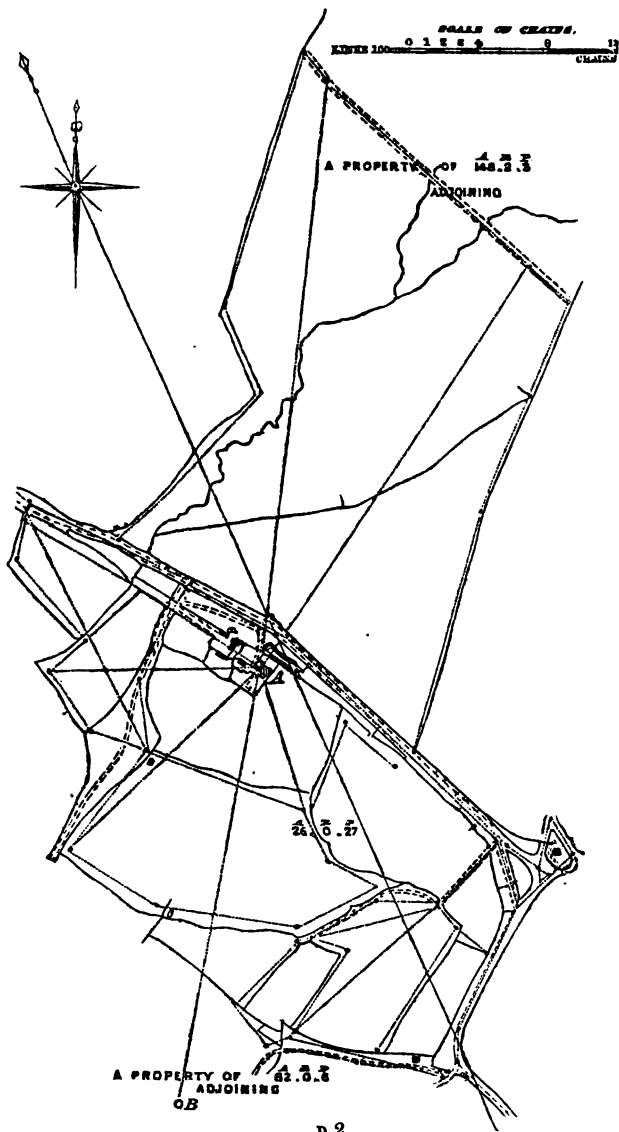
easily acquired, and the calculation will stand as under : —

Trapez. I.	205		In Trapez. 1.	175	1035	
	559			437	306	
	2)764			2)612	6210	
	382			306	3105	
	742				3.16710	
	764		In Trapez. 2.	274	507	
	1528			286	280	
	2674			2)560	40560	
	2.83444			280	1014	
					1.41960	
Triang. II.	878	2)196	In Triang. 3.	2)394	453	
	98	98		197	197	
	7024				3171	
	7902				4077	
	.86044				453	
					.89241	
Triang. III.	2)120	251	In Triang. 4.	2)83	888	
	60	60		41	41	
	.15060				888	
					3552	
					.36408	
Triang. IV.	2)122	355				
	61	61				
		355				
		2130				
		.21655				
Triang. V.	2)135	418	Trapez. I.	2.83444	Trapez. 1.	3.16710
	67	67	Triang. II.	.86044	"	2. 1.41960
		2926	"	III. .15060	Triang. 3.	.89241
		2508	"	IV. .21655	"	4. .36408
		.28006	"	V. .28006		5.84319
			"	VI. .08964		4
Triang. VI.	2)54	332		4.43173		3.37276
	27	27		4		40
		2324		1.72692		14.91040
		664		40		
		.08964		29.07680		

	A.	R.	P.
East Field . . .	4	1	29
West Field . . .	5	3	15
Total . . .	10	1	4

Had more fields been wanted, we might have proceeded from any one of the stations into the adjoining fields, always closing upon a former station, if possible, or upon some gate before determined by an offset, as in the close at the gate L; but if a great number of fields are to be surveyed, it is better to go round several first, always marking where other fences join into those we are surveying, as at M, and these will serve as points of departure, or to close upon when filling in the interior of what we have gone round.

Persons who remain long in one place are very frequently called upon for a small survey, to which additions will be required at a future time, and have no opportunity of making any general arrangement to secure the accuracy of the different parts when joined. The opposite plan has been prepared to show a method of avoiding inconvenience on such occasions, and has served for hundreds of acres on a 4-chain scale for content. At A, an old chimney was seen through the trees while surveying a piece of about 80 acres, consisting of many enclosures, and intersected upon the possible chance of future utility: some years afterwards about 50 acres was required to be added to it, and commenced at a point in the first survey; as soon as the chimney was again seen it was intersected, and as often as it presented itself, thus proving the work at any one of those stations. Again, a few years afterwards, near 150 more acres were to be added, and again beginning at a former



station, the same method attached this piece to what had been done before.

Under similar circumstances a survey of many square miles on the 8-inch scale was performed, and at a future period a similar quantity was added, proving at every place where an intersection was taken to the central object; but it must be observed, that most of the measured lines were straight for half a mile or more. The intersections were sometimes above two miles distant, and back angle surveying practised in both instances. This is called working around a point.

Now, when correct points are dotted down upon any scale, by a similar proceeding we prove the work as often as one or more of them can be seen; and as they are seen indiscriminately, they afford a system of tests in all directions, which renders accuracy undeniable.

CHAP. IV.

USE OF PLANE-TABLE, FINDING STATIONS, ETC.

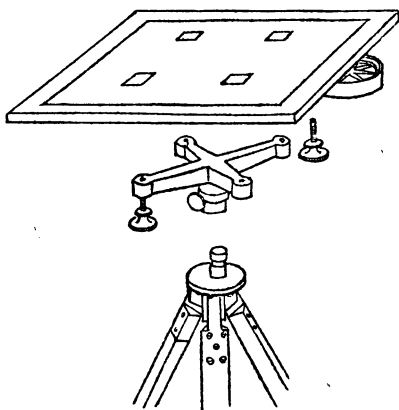
THE student has now been instructed in the use of the theodolite, chain, and protractor. We shall suppose him to have measured a large estate, and to have laid it down correctly upon rough cartridge paper, with the offsets, and every thing that is given in the field-book. There will be frequently many little things omitted which it would be a waste of time to employ a theodolite in doing; they are therefore commonly left for another instrument, which we use for drawing in the hills; it is extremely simple in its construction and use, and is called a Plane-table. We sometimes use this instrument entirely for planning, but it is not so exact as the theodolite.

THE PLANE-TABLE.

It is made in a variety of ways, most of which are inconvenient and heavy; nor can it ever be put in competition with a theodolite, even when in its most complicated form. To make it really useful, we must reduce it to the most simple state possible, — make it as light as can be, consistent with strength and steadiness, and then it becomes one of the best and most convenient instruments we have, to fill in

what remains of a survey after the other work is finished, and also to draw the hills.

Thus simplified, we have drawn it, separated into three parts, — viz. the table, the brass-work, and the upper part of the legs.



PLANE TABLE.

Its principles will be easily understood in the following description of the use we make of it in the field: —

A portion of the sketch being placed upon the table, the rim is shut down upon it and fastened by buttons underneath. It will be sufficiently stretched without wetting the paper, which would alter the scale of the drawing; and it would no longer fit the plot, or the part of it where it was taken from, if we cut up the original plot for sketching, and this should never be done unless to save time.

We now take it into the field, and place it upon some former station, or at one end of some long line, exactly as we do the theodolite, only that we level it by the legs alone, for the compass-card is a good substitute for levels in this case, where the greatest accuracy in placing the instrument horizontal is not required: we can judge very nearly by the card, which will touch the small point placed over it in the box, unless it is so nearly horizontal as to clear it while spinning round, as it generally does when first set down.

When we are satisfied that it is very nearly horizontal, the clamping screw is to be released, and a plane ruler to be laid over the corresponding station upon the plot, and any other convenient object within sight, or along the line we have supposed. Now we hold the ruler fast, and turn the table about until the distant object, or the end of the long line, is seen at the other end of it, when the eye is applied at the distance of about eight or nine inches from the nearest end; we now know that the figure upon the table and that on the ground correspond, or that every line on the latter is parallel to its representation on the paper. When the needle has ceased to vibrate, the degree that is under the small pin before mentioned should be noted on our paper, and it is obvious that at any place we please the table can be placed parallel to its first position by adjusting it to this particular degree of the card, supposing the needle not to vary in the mean time; yet this trusting to the needle may be avoided in every case where

two objects present themselves, both of which are marked upon the plan, (always preferring those which were fixed by actual measurement or intersection,) if we adjust the table to them as at first, — and the needle becomes a check upon the work, as it is in the theodolite.

It will readily occur to the pupil that every two points upon his survey, whatever objects may lie between them, will in fact be the extremities of a base equally true as if it had been actually measured; and the table being placed upon such points, and adjusted as before taught, we obtain intersections to all objects of which it may be necessary to find the place. This is done by placing the leg of a pair of compasses upon the station and the edge of the ruler in contact with it, turning the ruler as upon a pivot until the object to be intersected is seen along its edge, and then drawing a fine line by the same edge. This being done from three different places will be found very exact in most cases, although there is in strictness an error of the same kind as that mentioned in treating of the theodolite when it is not exactly over a station. The remedy for this, in the complicated plane-table, is by making the ruler or index a real parallel ruler, by fixing it to a pivot in the centre of the table, and then ruling a parallel to the line of intersection wherever it may be wanted; but when we are only sketching hills, or fixing objects which are not very small, in the filling in a survey, this nicety is of but little consequence: for the table being but small, the difference occasioned by

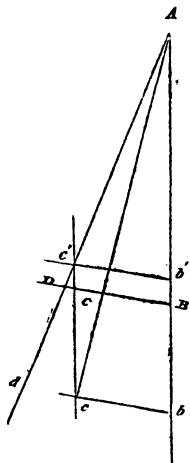
this parallax of the instrument can never make many inches error in the place of an object, and upon almost all scales in common use, this magnitude is but a mere point: if, therefore, we wish for that degree of accuracy, we must either do every thing with the theodolite, or use the complicated plane-table, and be satisfied with its multiplied apparatus, unwieldiness and unsteadiness, although that is a much more fruitful source of error than what we have just mentioned.*

Thus we may join up any unfinished lines or buildings that were left in the surveying to avoid short stations, or for other reasons; but the converse of this method by intersections is of most service, and hence it constitutes almost the sole use of the plane-table in sketching hills or finishing any work, and even that which is performed by this instrument only.

We have shown how to set up our plane-table by the needle alone, after having first ascertained its direction, when the figure represented and the natural figure are parallel. This, then, is done at any place where we can see two or more well-defined objects, the places of which are known to be well ascertained; and now, as the two figures are necessarily parallel, it is evident that, considering any two objects on the plan as a base, it is indifferent whether we go to them and intersect a third object, or whether, being at that third object, we draw the intersections back again towards ourselves: therefore this problem,

* If the angular point on the paper is placed over that on the ground, the parallax disappears altogether.

which is called finding our place, or *interpolation*, consists only in this — that having set up the table on the top of a hill, or elsewhere, and adjusted it by the needle, we place the ruler over some distant object, and its corresponding place upon the plan; then drawing a line towards ourselves, it is obvious we must be somewhere in that line, and the same will hold good of any other object. If, therefore, we repeat the same by another station or object, the intersection of these two lines determines the place where we stand. If a third object can be found, it is advisable to draw also a line from it for greater certainty, and it is, therefore, seldom omitted, although, theoretically, two lines would be sufficient. True as this is in principle, the practice depends much upon the nice adjustment of the needle, without which it becomes erroneous; it is therefore necessary always to pay particular regard to this circumstance; and it generally happens that several lines will intersect each other in the same point or place of the observer. Two new stations may also be found by the plane-table at one operation, whenever local circumstances favour it, (see diagram): thus let A be one of the points given by the sketch, and let it be required to find the situation of b' and c' , or b and c . Take the bearing AB; assume the point B upon it; and



take the bearing of D , the latter being to some object within reach, the situation of which we desire to know, and draw BD : pace the distance BC , and lay it down upon BD ; set up the table at C , and having adjusted it by the needle again, take the bearing of A , producing the line as far as may be necessary. Now this line will generally fall on one side of the point C , but it will always form with the bearings AB and BD a similar triangle to that we are seeking; and as we know the length of one of its sides BC , we have only to draw cc' parallel to AB , which will cut Ac or Ad in the point required; another line cb or $c'b$ parallel to BD , will give the other required station, or that we assumed at first. The greater the radius of the needle, the more accurately it may be adjusted. In angular measurements this is always of great consequence, because in long distances the errors become considerable: small needles should, therefore, not be used. A compass-card is preferable to a simple needle, because it sooner ceases to vibrate, and thus saves time; and it is also a better substitute for levels.

The Germans have a method of finding their place without depending upon the needle; but it is so prolix, and after all depends upon an exact assumption of two lines, in a figure rather complicated, that we have found it better to trust to a good needle, which experience has shown to answer every practical purpose, than to waste time in drawing figures only to last for a moment, and then to be rubbed out again.

Whenever a line is wanted to complete the junction of roads, fences, &c., the latter or former method may be resorted to. The last is most commonly used, as before stated; for, generally speaking, we have only to find our place and get intersections from thence, or the bearings of streams, fences, &c., between points already known, in order to add every thing necessary to the completion of our survey. In drawing the hills, interpolation, or finding our place, is every thing, as will be seen hereafter, when we come to that part of the subject.

It may not be improper to remark, that the theodolite is admirably calculated for interpolation, and it will frequently save much time and trouble by beginning from a place so found.

Practice will soon show the pupil the most expeditious and certain use of his plane-table, if these instructions are properly attended to; and he may make a sketch with considerable accuracy, upon a small scale, with this instrument alone. By first measuring or pacing a base, and intersecting objects from each end, avoiding acute intersections, unless they can be confirmed from better places, these become points from whence he may obtain other intersections: these he fills in by the eye, always preferring intersections, or finding his place, to pacing by a zigzag course, when it can possibly be avoided; and if points have been carefully fixed with the sextant, or otherwise, he need only dot them down on his paper, and having once found the direction of his needle, as before taught, the whole may be done both expedi-

tiously and correctly, and has long been constantly practised over many square miles.

We must observe, that in many cases where a distant object is much above or below the horizontal line, or plane of the table, the mere edge of a ruler is not of sufficient thickness to observe by. The index of the old plane-table has two sights, with slits and perpendicular wires to meet this exigence, and is certainly preferable* on that account; or we may erect two perpendicular brackets, (as in the annexed figure,) the edges of which will serve as well; for, without them, we must *suspend* a small plummet between the eye and object, and place the ruler by the help of it; or we must note some object in the same perpendicular plane as that we are intersecting, and use it instead; some tree, stub, bush, post, or other object generally presents itself, so as to occasion very little uncertainty in this respect.

It is very likely that in a perusal of the foregoing and following pages, the cautions against error in some cases, and the apparent toleration of it in others, may appear unaccountable: the following brief remarks will, it is hoped, remove every appearance of incongruity.

* But then it requires parallel plates and a level.



All instruments are in a degree imperfect, and, consequently, the operations in which they are used are liable to small errors: these, where they are likely to accumulate, must be carefully guarded against. By a good practical application of the instruments, they may be, in many cases, much lessened, and in others entirely removed: in using very large instruments, this is a circumstance requiring constant attention on the part of the observer. Now the errors which are of most consequence, arising from wrong angles in the principal parts of the work, are of this class: for if the greater points are wrong fixed, error will accumulate; but, by paying much attention to this circumstance, and not deriving the minor points from one another, when it can be avoided, but always from the first-mentioned, a little error in those minor points will not be perceptible; and, as each has only its own, it is divided and not accumulated. Hence, the principle is developed, which has been followed in these instructions, namely, that all imperfections of workmanship in instruments, which are unavoidable, should be divided in such a manner as, in most cases, to vanish entirely in the drawing; and this is the great principle in the use of all instruments whatever. It will be recollected, that we have especially noticed angular errors, because the subtences of those angles increase so much with the distance *, while a local error of a few inches, or even a foot, is not material in a case where it can be propagated no further.

* See Appendix.

To those who are accustomed to the use of large instruments, all this is well known; and that, although mathematical solutions are undoubtedly true, yet, when practically applied, the greatest skill is often necessary, in order to avoid false results from the imperfection of instruments. Fortunately for the surveyor, those he uses are comparatively small, and more firm in their construction. He also derives another considerable advantage from the scale of his drawings, where, as we have shown before, a magnitude sensible enough upon the ground is very insignificant when exhibited upon paper.

CHAP. V.

GENERAL OBSERVATIONS UPON SURVEYING, AND INSTRUCTIONS FOR FILLING IN TRIANGLES. — INVESTIGATION OF A TRUE SCALE OF SHADE.

THE student who understands what has been before taught, is perfectly qualified for filling-in the triangles of a large survey, as that of Great Britain and other countries. In those which are laid down on so small a scale as two or four inches to one mile, and where the country is not full of steep acclivities, the reduction to horizontal bases cannot be separately appreciated, but the lines will overrun; and the practice before spoken of must be resorted to when necessary, or the survey cannot fit the points which have been obtained with much care as a check upon its accuracy.

The number of offsets will be but few, for they cannot be shown on so small a scale: this much abridges the labour, and the number of entries in the field-book are proportionably lessened.

Supposing that a few trigonometrical points have as usual been given to a surveyor to lay his work down to them, he must take an opportunity of going to one or more of them before he begins plotting, and take the bearing by his theodolite compass

of some other very carefully. Now by this he can lay down the meridian of his own particular theodolite, which will in general differ a little from that given by another theodolite from the same place and to the same object; and whenever any other instrument is taken into use, the same operation must be performed with that, and then the work done by all will accord, as it ought to do; for as the magnetic meridians of different theodolites generally differ a little, from errors of centring, or some local attractions in the instrument, that difference must be tried upon some known bearing within the survey, and those used for plotting being adjusted accordingly, there will be no occasion for twisting the work; and thus the work of several instruments may be laid down on the same paper without error, instead of being drawn on separate pieces and joined afterwards. In two theodolites that were tried, one needle gave $323^{\circ} 15' \text{ N.W.}$, the other $320^{\circ} 38' \text{ N.W.}$; difference $2^{\circ} 37'$. Similar instances have been found in other instruments.

Many persons do not use this precaution, but it is highly satisfactory to have done so, when we see the accordance it produces; and in the writer's opinion it is absolutely necessary in correct work. It is true that very often the points given are not conveniently situated to admit of this method being used, and in that case it is preferable to use the corrections for eccentricity of stations, or any indirect method, to merely trying the work after it is plotted, by applying the points, as is more frequently practised.

Trigonometrical surveyors would do well to mark carefully the points they have chosen, instead of suffering them to be lost, or giving a verbal and insufficient description of them, which creates much uncertainty and great trouble to the surveyor who fills them in.

We have led the student through that part of surveying which may be termed linear, and which requires only good instruments and care. We must now recal his attention to the problem with which we set out, and state that there are two methods of showing the solid after these lines have been drawn: one is by vertical illumination in parallel rays; the other, falsely called light and shade, is by oblique rays issuing from the left hand upper corner of a drawing, and also parallel to each other. It is evident, that by the first method the light which falls on the slopes of hills will be much lessened as they approach to a perpendicular. The exact determination of its quantity, though not difficult, has occasioned much discussion when plans or maps are destined for military purposes; and we cannot do better than investigate the subject considered abstractedly from all reflected light whatever.

Let parallel rays fall perpendicularly upon the horizontal plane $A B C D$, and it will receive its maximum of illumination. (See diagram.)

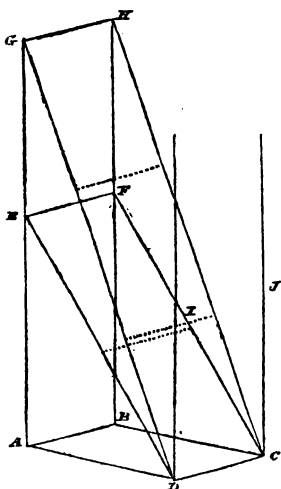
Suppose now another plane $C D E F$, whose side $E D$ is twice the length of the first plane, to be placed at such an angle as that E and F shall be respectively in the perpendiculars $A E$ and $F B$: it is evident

that this second plane, inasmuch as it contains double the superficial space of the first, will receive but half the quantity of light upon any given portion of it, that was received upon a similar portion of the first.

Suppose, again, that another plane $C D G H$, whose side $G D$ is three times the length of the first plane, be so placed that G and H are in the perpendiculars A and B respectively; it is also evident that this last plane, because it contains three times the superficial space of the first, will receive but one-third of the quantity of light upon any given portion of it, that was received upon a similar portion of the first; that is to say, the light decreases inversely, as the superficies of the planes increase, by increasing their angles of elevation; or twice the plane of the base receives but half the light, and three times the plane of the base receives only one-third of the light, and not twice the angle, one-half of the light, &c. as generally stated.

And the same will take place let the breadth of the planes be whatever it may, if the other dimension alone is augmented.

The decrease of light is thus easily made out; but



if this be admitted whereon to establish a principle of shading, we must take the converse, as it is the shade with which we have to perform our operations, whereas in nature the shade is a sort of negative, being only either a total privation or a decrease of light. A plane, perpendicular to the horizon, if perfectly smooth, could receive no light, and the shade in this case becomes a maximum; we must now determine what proportion it bears to the light in the intermediate degrees of obliquity.

We shall first consider the case where the superficies of the plane is doubled and the light reduced to one-half, answering to an elevation of 60° . Now, as all perpendicular planes are alike deprived of light, without regard to their extent, we may take the plane $DIJC = ABCD$; and supposing shade to be of a positive nature, as indeed it is in practice, imagine this plane $DIJC$ to extend itself until equal to $CDE F$, and to fall down to an angle of 60° : the shade will now be extended over twice the space it originally occupied, and consequently its intensity will be reduced to one-half; and so also is the light which falls upon the increased plane: hence there is an equal portion of each.

If $DIJC$ be extended until it is equal to $CDGH$, or to make an angle of $70^\circ 31' 44''$, the shade will be dispersed over three times its original space, and so also will the light; each will, therefore, be only one-third of its original intensity; and this reasoning obviously holds good in all cases whatever. It therefore follows that both the light and the shade

decrease in proportion of radius to the cosines of the angles at the base, or their equals the angles of depression at the top, because the sloping planes are increased in that proportion when compared with the horizontal planes beneath them: or, we may more simply say, they increase as the secants of the angles of elevation to the radius AD or BC .

If we consider the whiteness of paper to represent the maximum of light, or that on a horizontal plane, and the deep shade of Indian ink as the positive representative of a total privation of light on a perpendicular plane, then the law of diminution before deduced will give us the intermediate shades.

If a given quantity of liquid and intense shade be capable of covering a given superficies, suppose a square inch, then it is evident that by adding an equal volume of water it will cover two square inches, or double that space; for the colouring matter, being spread over twice as much space, will lose one half of its intensity: for, as water communicates no tint of itself, it may be considered as the representative of light, and must be used to spread the shade over a greater space: this, then, is the proper shade for a slope of 60° . If we add to a like quantity of water twice its volume of shade, it will now cover three times the original space, and its intensity will be increased one-third: it is, therefore, the proper shade for $70\frac{1}{2}^\circ$ nearly, and so of any other between 0° and 90° .

The following table contains the numerical values of the slopes for every 5° up to 45° ; below 5° the

difference of shade is hardly perceptible, and beyond 45° the ground is generally so broken as to present a rocky or craggy appearance, which must be left to the skill of the draughtsman, who must draw its foreshortened appearance from real studies, if he wishes to excel:—

Elevation.	Value of increased slope.
5	1.00382
10	1.01543
15	1.03528
20	1.06418
25	1.1034
30	1.1547
35	1.2208
40	1.3054
45	1.4142

From this table it appears, if the plane $DIJC$, equal to the plane of the base $ABCD$, be taken as unity, that at 5° of elevation the sloping plane is greater than the horizontal base, in the proportion of 1.00382 to 1: there is, therefore, only .99618 of light left upon it, the shade being diminished from absolute black, or 1, to no more than .00382. In like manner the light also is in effect reduced .00382 parts from unity, leaving as a remainder .99618: hence, if the water necessary to cover this slope is divided into 100,000 parts, there must be 382 parts of ink added to it to enable it to cover the increased plane: or, in other words, to obtain a shade proper for that slope; for, while the proportion of water to shade is kept, the quantity is indefinite.

At 10° , 1543 parts of ink added to 100,000 of water will give the true shade, and so on. Now, a

slope of 20° is very difficult to ascend without following a zigzag course, as most persons must have experienced; and to express this degree of elevation, we ought to use little more than six one-hundredth parts of shade to one hundred of water. It is, therefore, plain, that the practice of exhibiting such very dark shading on plans, as is generally followed, is very erroneous; and although all allow the present plans to be overshadowed, yet few seem to be aware of the true quantity that should be used; but from observations carefully made upon irregular solids, illuminated at every possible angle, and the theory we have deduced, there can be no hesitation in admitting, that the theory allows, in some cases, even more shade than would be visible under the same assumed circumstances; because, in nature, the light reflected from one oblique plane to another considerably reduces the shade which theory points out to us, and, therefore, it has been thrown entirely out of the question in the foregoing investigation.

We have now developed the principles which all profess to be guided by, and certainly no one correctly follows: for although we cannot estimate light by absolute quantities, yet its ratio, under all possible circumstances, is perfectly appreciable, and the quantity of the substance employed to produce shade may be computed to any degree of exactness; therefore, if every plan could be drawn upon these principles, no difficulties would occur, as they constantly do when several parts drawn by different persons are

to be united into one large drawing*; for as no certain data at present exist, whereby to determine the relative values of slopes, if we except some few ill-contrived characters to be drawn with pen or pencil, and which never can express ground naturally, there is always a chance of mistaking when several parts of a large survey are to be joined together by a person who receives them from their respective surveyors.

It might seem, therefore, that these principles could be made practically useful to learners by accurately constructing a scale of slopes for every 5° of elevation, according to the law before deduced, without regarding the effect of reflected light, and referring to this as a standard upon all occasions; and that a habit of conforming to this scale might by degrees be induced: but the vast labour of attending to such a procedure would never be compensated by the accuracy thereby attained, for the greatest skill and nicety alone could insure success.

Experience daily shows that very seldom, even when in one colour, and when skilful persons are employed, a drawing is copied faithfully as to strength; and the difficulty which occasions this deviation would be still greater, if the thing to be copied were itself constructed with such nice and discriminating care, as a strict adherence to these principles would demand,

* This inconvenience is now remedied by using a clinometer or quadrant with a plumb-line, card or brass will do, and the actual angles of the slopes are written on their respective places, in all sketches of the first or second division.

and, besides, the slopes of hills are not mere inclined planes, but curved surfaces of the greatest variety ; and, hence, the application of these principles is of too intricate a nature ever to be admitted in practice, for we should then need an infinity of elevations in every part of a survey ; and what is now a laborious operation, if conducted properly, would then be rendered infinitely more so ; and there can be no doubt of producing very perfect plans, if only the principal slopes are regulated nearly upon these principles, and drawn as usual by repeated shades laid upon one another, and blended together in a proper manner.

We should not have entered upon the foregoing investigation, if it had not been that most books upon topographical drawing set out from a hill scale, which is generally assumed at the discretion of the authors, but, apparently, without much attention to truth ; and certainly, as we have said, when different marks or characters are employed to represent different angles, the endless variety of curvature in ground utterly defies the application of them.

We have insisted upon the necessity of drawing hills paler than most persons like to do, from having frequently seen that draughtsmen exhaust their scale of shades long before they have produced an effective representation of their ground.

We should further consider, that above 45° , and often below it, the slopes are either so broken or so rocky as to admit of a more picturesque representation than those of a gentle nature ; and this is what most draughtsmen are fond of doing, — the tameness

of a succession of gentle slopes being generally little suited to the taste of any one who has the least pretension to the name of artist.

Although we have now obtained the law of diminution of the light, and shown the impracticability of applying it with precision, we must observe, that much attention should notwithstanding be shown to the distribution of shade, and some common agreement at least respecting the greater slopes be well understood; for the portions of an extensive survey are often united by a person who has not seen the ground. If, therefore, the persons who draw each portion endeavour to give them effect by overloading them with shade, much ambiguity will arise, and the pieces cannot be joined properly; for example, each piece being usually bounded by a road, if that road crosses a hill, and the hill is on one piece shaded darker than its corresponding part on the other, then the person who unites them has nothing left to guide him but theory, and that is not entirely sufficient, especially in its present imperfect state, the difficulty being moreover considerably augmented, while the identical character of hills is frequently lost in a mannered style of drawing.* But if ever the individual character of hills as to exterior, and the usual angles of their slopes, when of like kinds, should be well established, then much, if not all, of this uncertainty will disappear, even should the drawings be defective in the shading.

* The clinometer, properly and effectually used, is the best way of avoiding any difficulty of this kind.

CHAP. VI.

METHOD OF DRAWING HILLS AND OF SKETCHING THEM ON THE GROUND; OR THE REMAINING PART OF THE SOLUTION.— ILLUSTRATION OF THE LAW REGULATING PENCIL TOUCHES AND SHADING PLANS. — OBLIQUE LIGHT CONSIDERED.

THERE are two methods of describing hills, both in the field and in the drawing-room; one called the vertical, and the other the horizontal style, which are, or rather should be, peculiar to the pencil or pen; for there is no absolute necessity for having recourse to any touches on an Indian ink plan, if the irregularities of surface are properly described by shade.

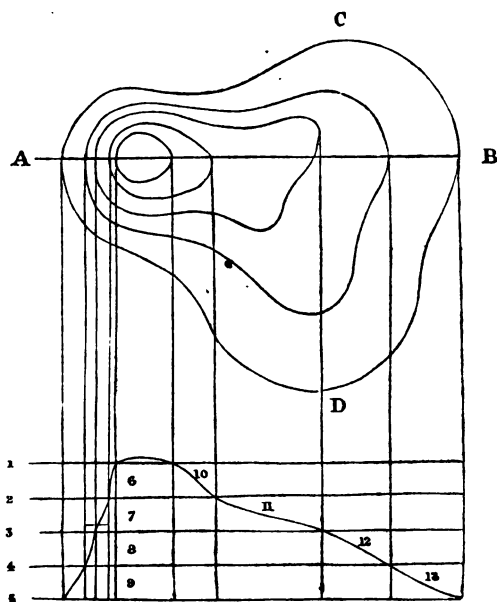
In the first method the shade is formed by strokes of the pencil or pen radiating from or converging into any curved part of a hill, according as it projects or re-enters: they are supposed to describe the same course as water would do, if it could trickle in streams down the slopes; and we hardly need observe, that they are darker or lighter according to the steepness of the slope.

The other method has the shade formed by lines parallel or nearly so to the horizon. It is much more easy to apply, and more natural than the former, although, on some kinds of ground, a judicious

draughtsman will, perhaps, employ the first, or a mixture of both; experience in sketching will point out the use of either or both, as in craggy ground, rocks, &c. A specimen will illustrate this completely, and will be found in *Plate II.*, which has a representation of the same hill by both methods, and also a piece of broken ground with the two styles intermixed.

The horizontal style has, perhaps, some claim to particular notice, from its easy application in sketching, and the facility with which it may be demonstrated and acquired by the following experiment:—

Procure a stone somewhat resembling a hill, as may frequently be found, and a box that will just hold it, with a small space around it: bed it in clay placed in the bottom of the box, through which there should be a small hole and a plug; fill the box with water stained with Indian ink, and let it off by means of the plug, about a quarter of an inch in depth at several times, allowing sufficient intervals for the fluid to stain the stone in that plane it has fallen to at the last abstraction. These stains will present a series of horizontal lines all over the surface of the stone; and if we then examine the stone thus prepared, looking down upon the top, we shall see that the steepness and flexure of its sides will be accurately marked with a proportional number of horizontal lines at such variable distances as will very properly express it: thus, in fact, we obtain a sort of scale of the relative steepness. If A B C D (in the opposite diagram) represent the base of a stone,



and the remaining lines be traced horizontal around it at equal heights, then letting fall perpendiculars from the intersections of the section line A B with the above lines, and drawing the parallel lines 1, 2, 3, 4, 5 at the required heights, we have but to unite the diagonals from 9 to 13, and we obtain a correct section anywhere we please. This is much practised on the Continent, and called a system of contour lines; but in this as in the other hill-scale, the rule deduced is impracticable without immense labour, yet it will nevertheless help us considerably in judging of the direction of our lines when sketching upon

the horizontal principle, if we practise drawing from stones so prepared ; and the student who begins in this manner will very soon approximate to the truth in sketching from nature, to which it is the best introduction, for it forms the hand very soon and very correctly. We may also strongly recommend the learner to draw from plaster casts of figures, or good chalk drawings of them, which will materially assist him : good models of hills or mountains would be still better, but they are by no means common productions.

Whether a drawing is to be illuminated vertically or obliquely, it is always sketched as in a perpendicular light. We shall now proceed to give the student some instructions how to perform this in the field. We shall suppose that he has placed and adjusted his plane-table on the feature (see figure in page 54.) facing the front of it, and that he sketches in the horizontal manner. He is to examine the feature where he stands, attentively, and ascertain the boundary, or where it begins to slope, and sweep the first line around it, or several shorter lines, as he thinks proper. He then is to make curve lines down the front, according to the degree of steepness, and next draw the sides in the same manner, properly distinguishing by the number, faintness, or boldness of the touches, the relative steepness of the slopes. He now, by means of his plane-table, finds his place upon some adjoining feature, where he repeats the operation, and so on through the whole, observing that the first piece he has sketched becomes a standard for the

whole in point of strength of shade: he will, therefore, do well to occasionally measure an angle or two, where the slopes are long enough to admit of it, with a clinometer, until he can by practice do without it.

While sketching the stronger features, or main hill, he should constantly determine by intersection the remarkable points of the under-features, unless he prefers finding his place when upon them; and he must remember that, when on these lower elevations and surrounded by higher hills, he cannot always rely upon points being visible by which he may ascertain his situation; they must, therefore, be judged by the eye with reference to those which are near, and practice will soon teach him this. The surveyor should never quit a place he has found correctly, until he has made every intersection likely to be of use to him, and in fact done all that can be performed there, or he will sometimes be obliged to return again, and thus lose much time. We have here supposed an open country and small features; in an inclosed country with larger features, where the roads, fences, streams, &c., are numerous, they will almost determine the positions and even the form of the hills, and the plane-table becomes a mere rest for the arm; on such occasions, therefore, a copy of the rough plot is put into a sketching-case, upon the back of which is a contrivance to hold it by the corners, the hills being sketched in as the roads and rivers direct us.

If we have occasion to determine the position of a summit or other part of a hill, we are almost certain

to find that lines drawn through particular objects would intersect each other, near where we stand, and a very slight alteration of position will find the place, exactly : we accordingly use this method, and draw the hill as it may happen to be with respect to a place so found.

There are many little undulations in the sides of most hills that cannot properly be called features* ; these should be particularly marked, as they stamp a character upon them. Indeed, if the figures of hills are ever destined to be digested into a classification depending upon their external form, every lineament should be carefully noticed.

The junction of under-features, and attaching them properly to the larger ones, is commonly difficult for beginners. Nothing can give a better idea of it than the stone before mentioned, if it is sufficiently irregular to present knobs almost detached from each other, which in some measure resemble those appendages to a natural hill.

When the manner of applying the lines is well understood from the model, the horizontal system of sketching is really nothing more than the shading which is used in pencilled landscapes, applied around curves, instead of being in lines always parallel to each other. The intensity of shade is formed absolutely in the same way by darker strokes, or by crossing those already drawn with a small degree of obliquity ; and the vertical system is exactly the converse of it, the touches being always perpendicular

* This is admirably illustrated in the engraving, performed by the Anaglyptograph.

to a tangent to those curve lines, let the curve be whatever it may.

Little more can be said upon sketching: practice alone will enable a person to do it, and he will soon become expert, if he will but draw what he sees, taking nature as he finds her, copying closely the peculiar character of cliffs, as well as hills, and taking nothing for granted. He may be forgiven a little stiffness of expression at first, for by practice he will acquire a certainty of eye and hand, and freedom will inevitably follow.

The ground now being all drawn, and every thing done which had been left out in the survey, there remains yet to trace the sketch upon the plan, and, as before said, it may be illuminated vertically or obliquely as may be thought proper.

The hills should be sketched lightly but carefully on the fair plan, for much evidently depends upon this: it would be fruitless to employ so much care in the field, if we did not follow it up to the last.

A light tint of Indian ink is to be distributed freely along the tops and most elevated parts of those hills, which are the origin of the under-features, and softened down into the ravines with a brush and water, then upon the next series of levels, and so on until the last under-feature has been shaded; whatever parts still want strength are again to be similarly shaded, beginning a little below the first tint, and thus until it is finished. If the ground or any part of it is irregular or rocky, a rougher shading must be disposed upon it, as, to the judgment of the draughts-

man, may seem to convey the best idea of such local circumstances. It has been usual to finish drawings by vertical or horizontal touches, after they have been sufficiently shaded, in order to give them spirit, or show the direction of the slope; for one or the other of these reasons is occasionally given—with us it is a questionable matter whether it is necessary to retain either of them.

A drawing with the smoothest shading will certainly want spirit, but this tameness belongs to the subject, and false ideas are conveyed by introducing rough shading where it evidently ought not to be. There is a certain freedom of touch and harmony of shade which will describe smooth ground perfectly; in this case, the shades must be increased by less increments than in the bolder subjects. The student will do well to use only such vertical touches as are necessary to show the marks of torrents upon the slopes of mountains where he finds them; and only such horizontal touches as describe correctly the rugged steps by which other hills or mountains are characterised. This is, indeed, drawing from nature, and necessarily preferable to either horizontal or vertical touches, used merely for effect; accordingly, it is fast gaining ground, and long has been, amongst those who are not prejudiced in favour of former systems. There used to be as much time consumed in disposing of those touches, as in the previous shading; and in the opinion of learners, it was a universal remedy for a badly-shaded plan, by which means they were rather retarded than benefited by

its adoption, since the shading was always of minor importance with them. A free use of the brush will prepare a surface by leaving lighter parts upon any slope of the proper shape; these we convert into stones or other ruggedness which it may be desirable to produce.

If we prefer an oblique light, custom seems to require that it should fall upon the horizontal plane at an angle of 45° , and from the left hand corner of the drawing. If none of the acclivities measure so great an angle, then it is obvious the light will graze them more or less according to their steepness, and a full shade capable of producing effect can nowhere exist upon such a plan. Since, therefore, this disposition of the light is adopted solely for effect, may it not be reasonable to adapt the angle the light makes with the horizon to the nature of the ground to be represented?

In this case, it will be always less than the greatest angle of slope which faces the opposite part of the drawing to that from whence the light issues. A flat country, whose slopes do not exceed 20° , will, therefore, require the luminary to be elevated about 15° , and in mountainous districts it may be taken as high as 40° . Under whatever angle the light is supposed to enter, the draughtsman must study well its arrangement, and should be acquainted with the angular elevation of his principal slopes at least; the relative value of the inferior ones he may infer from local circumstances, and his own observations while sketching. He must begin, as in the vertical illumi-

nation, to lay pale tints upon all those slopes that are turned from the light. These must be softened into the ravines and valleys wherever the curvature of the surface requires it, leaving the slopes, opposed to the light, white. He may then work up the slopes as his judgment directs him, considering well how far the light will prevail over the shade in those hills or features which lie nearly in the same direction as itself. The study of models is a good method of acquiring a knowledge in the application of oblique light; and although we cannot see hills from overhead, as we are supposed to do in a plan, yet the intelligent draughtsman will collect many valuable hints from a careful observation of nature herself under all circumstances of illumination. A little rugged hillock will frequently towards sunset produce such a beautiful illustration of the principles of light and shade, as to an eye accustomed to study drawing will be invaluable.

But there is another consideration to enter into — the system of oblique light. There are three modifications of light upon all natural objects when illuminated — the direct, reflected, and the perpendicular: of the first enough has been said; of the second it remains to say, that it would introduce an additional difficulty were it to be employed; but of its effect upon the drawing we shall now inquire. Wherever any slope opposed to the direct light was inclined to the horizon at such an angle as to be capable of reflecting it back upon an opposite slope, it would diminish the shade upon that opposite slope, and in a

coloured drawing would produce a warm tone of colour; for the reflected is generally warmer than the original or direct light. This could only happen in those cases where a slope was more inclined to the horizon than the direct light, and then if the slope opposed to it was not very near, the reflected light would be much weakened before it reached it; still such cases, when they occur, should be noticed. The perpendicular light can exert but little influence from its weakness, being only those rays which are scattered through the atmosphere when it is cloudy, or sent down from white clouds floating nearly over head; or, in a cloudless sky, it is merely the blue rays from the sky reflected back upon the earth, occasioning grey, and, indeed, sometimes even blue shadows, by destroying the local colour where those shadows fall: thus it could only tinge the tops of mountains of a colder colour than the direct light; but in a drawing of only one colour this faint blue light should be expressed by a very pale shade, as it is customary in drawings of one colour, where, being confined entirely to light and shade, we have only that method of expressing a very faint tint, let the colour be what it may.

To return to our drawing, it must be observed that all these circumstances having been properly attended to, we must next consider the two remaining unshaded parts, namely the slopes facing the light more or less, and the tops of the hills; these must now be distinguished from each other, and in doing

so we must constantly consider the degree of obliquity to the direct light, as produced by their inclination to it, and also to the horizon, and they must be shaded accordingly, the last decided touch being given on the sides, the recesses of rocks, &c. &c. as may be necessary.

CHAP. VII.

TOPOGRAPHICAL MINUTIÆ, WITH FURTHER OBSERVATIONS
ON LIGHT AND SHADE.

THUS far we have only spoken of the roads, streams, and hills; but the towns, villages, woods, morasses, marshes, and other minutiae are to be attended to in proper order. That these things might be comprehended in one plate, and treated of by themselves, we have omitted to mention before that the woods which were formerly shown by little green dots, when upon the sides of steep slopes, were always productive of confusion, or, at least, indistinctness: for those slopes having been already finished, every thing else that could be done upon them must have that effect. Now, in order to avoid this, we suppose the trees and woods to be seen by visual rays making an angle of a few degrees with the perpendicular — they will then admit of a disposition at once picturesque and according better with the undulations of the ground than by the old method, particularly in a plan wholly illuminated by an oblique light (this will be better understood by reference to Plate V.); consequently we should arrange these woods either by pencilling or penning the outline with light ink, as

soon as the hills are sketched, and, in shading the parts that fall upon slopes, take care to accommodate our shading to the hill so as to preserve its form very perfect; this, however, is a little troublesome to learners, particularly when they have not previously acquired a knowledge of landscape drawing.

The villages will have been drawn during the survey and sketch; the houses may now be put in red, which is a colour never applied, except faintly, until the shading is finished, as it would be disturbed by the shade or water, and disfigure the drawing. The same remark applies to all soft colours whatever. When the scale is small, as four inches to the mile, they become mere small rectangles, and are sometimes distinguished by making the brick or stone houses red, and the wood or mud tenements black or brown. On larger scales they may be drawn neatly with a steel pen, and after colouring them, finished with a thick line on those sides where a shadow would fall, when the light is in the left hand upper corner: and when on very large scales, the roofs are drawn, instead of the mere rectangular site of the houses.

Upon all small scales we are obliged to enlarge some objects beyond what the scale will warrant: this remark applies generally, excepting to the hills; but on scales greater than twenty inches to a mile there is room for every object according to its proper magnitude: such distortion should, therefore, cease at that point of enlargement, and indeed in many cases long before.

It is a good practice to consider plans on different

scales as objects seen at different distances ; it helps us to a mode of generalisation, and much simplifies the drawing. Thus the rock of Gibraltar would admit of being drawn perfectly in character on the twenty-inch scale, yet, on the four-inch scale its minutiae would be blended together, and the rock would be expressed by little more than a simple shade.

By a similar mode of reasoning we admit the propriety of extinguishing the minor features of ground, and blending them in one general slope when we draw on so small a scale as one inch to a mile, or less ; for then it is impossible to find room for many of them.

We have now gone through the process of planning and drawing, according to the prevailing methods, and have also given examples of the minuter parts, many of which, it must be remarked, are always drawn as they would appear in an oblique light, although the vertical illumination is used for the rest of the drawing. Some observations will now follow upon the usual practice. It has constantly been found by experience that when rocks or mountains, having pointed or angular summits, like the roofs of houses, are illuminated vertically, their appearance is gloomy and unsatisfactory ; they have, therefore, been frequently obliquely illuminated, even when the flat country at their bases has been drawn in the other manner. In the vertical illumination, the tops of hills are always the lightest or unshaded parts ; and when the oblique light has been supposed,

they have also been left white. Now this is a manifest absurdity, because whatever angle of inclination be chosen for the light, there must be some slopes among mountains opposed to it, which will, therefore, receive more light than the tops themselves, the maximum of light always falling upon that slope which is most directly opposed to it, and not necessarily upon the tops of ridges. Hence, to bring the tops of mountains nearer the eye does not depend so much upon having them lighter, as upon a proper distribution of breadth and intensity of shade, as in nature, and throwing the strongest light upon the highest point; for as aërial perspective does not interfere with the measurable dimensions of objects, but only blends them together in broad masses, or softens them into good keeping, this is the proper place to introduce it for giving the best possible effect to a plan drawn, according to a false technical expression, in "light and shade," or, indeed, to any plan whatever.

In coloured drawings it is particularly conducive to beauty and perfection; for the lower objects being more massed together, and more grey in their general appearance, detaches them from the upper or those nearest the eye, for which all the warmth of light and strength of colouring is to be reserved. The minutiae, particularly those in the low grounds, should in all cases be in such keeping, as to strength, as not to spoil the masses of shade, and thereby take from the apparent height of the hills.

In mountainous districts, where the profile is angular, the vertical illumination compels us to give a

certain space to the tops, otherwise the two sloping sides, or rather their corresponding shades, meeting at the top, destroy the very light upon which, by being supported by shade on both sides, those summits should come nearer to the eye, or rise from the paper. Hence a white stripe or a light one being left to represent the top is quite out of keeping with the rest; and the same thing happens with pointed summits: this is the reason why the oblique light is preferred, and, indeed, almost forced upon us in such cases.

It might seem, from what has been said, that the oblique light would be preferred, and so it would but for its supposed ambiguity. It is easy, when depth of shade stands for steepness, to distinguish at a glance the relative value of slopes if correctly shaded; and it perhaps requires little more consideration, in some cases, to distinguish the same in the oblique illumination. Since it must, however, be confessed that no practicable method has been devised whereby the exact angles of slopes can be inferred from their proportion of shade, but barely their relative inclination*, (unless we lose sight of the true natural forms in our drawings, in order to gain that supposed advantage,)—we can, therefore, only find one really valid objection to oblique illumination, and that is, its difficulty of execution. But whenever the nature of ground shall have been shown in a tolerably perfect classification, and each distinction strongly marked

* Even this, we have shown, requires more attention than it is possible to devote to it, and, from their very nature, these angles defy all such attempts.

in a well-arranged set of drawings or models, the difficulties of reading it, as it is called, will cease; and, like all other arts, when sufficient improvements have been effected, it will be well understood by all those whom interest or inclination shall lead them to this branch of study.

We do not think it necessary to give particular directions for surveying woods, rivers, harbours, fortifications, towns, &c., because these are only particular applications of the general rules we have laid down: every thing can be done by bearing and distance, aided by intersections to the inaccessible objects as checks upon the operations. Thus, while surveying one bank of a river by bearing, distance, and offsets, we intersect objects upon the opposite bank, — lines between which may afterwards be drawn, and surveyed in the usual manner.

In level countries surveys may be carried on by the chain alone. To make this understood, we have only to refer to *Plate I. fig. 2.*, in which, if diagonal lines had been actually measured, so as to determine the three sides of every triangle that can be formed within the lines measured around the fields, we can evidently lay them down without having the angles; and passing into the adjoining fields from one convenient station, we do the same thing again, and close upon another former station. Thus we perform small surveys with a chain and a little additional measurement; but when they amount to some thousands of acres, the principle of subdividing errors must be applied. When the chain is used with very

little employment of instruments, a method is practised by some surveyors, of measuring lines of great length perfectly straight between two marked points, and thus forming triangles; we shall explain how to perform one such line, and then the rest will be easily conceived. We have already remarked, that the longer the lines, the more simple the figure, and the less chance of inaccuracy. If the bearing of this line be ascertained, and upon carefully measuring it more than once, marking every 5 or 10 chains, with the boundaries passed over, and every other thing that may be deemed necessary, it is easy to survey the enclosures on either side to about 10 chains on each, as required for parliamentary surveys for railroads, taking care to reduce all ascending or descending slopes if necessary, indeed noting every thing through the whole: but in this case permission must be had to cut through copses, fences, and all that can be perforated; but there remains something else to be thought of, and this is, when the line passes through a village, house, pond, wood, &c. In surveying by chain and theodolite, as we always go round every thing by bearing and distance, this circumstance will seldom occur; here the case is otherwise, and the obstacle may be cleared thus: turn off at a right angle to the line, and measure till beyond it, then again at a right angle, and again beyond this; another right angle will bring us into the original line by measurement, the second side measured being equal to the unmeasured distance, and if rightly done the bearing will be the same as at first; the

line is then to be continued as far as necessary by such means, unless the following is preferred: turn off at an angle of 60° , and having measured clear of the obstacle, again return to the original line at 120° , measuring the same distance; then the unmeasured distance will be equal to either of those that deviate from the original line.*

In Captain Dawson's Instructions on Parish Surveying, and in those who have copied them, excellent directions have been given; it is a pity that his recommendation has not been followed, for then, with its adjuncts, a real doomsday book would have been formed. Railway surveyors follow this plan of surveying, and when a curve occurs, the angle between the two lines comprehending it is carefully measured by an expensive theodolite, and laid down by a protractor, the direction of the road having been previously laid down provisionally by the engineer. If a large extent of ground is to be surveyed, and triangles based upon a principal line, following the same method throughout, and subdividing it into smaller triangles, no doubt can be reasonably entertained of its accuracy; angles are seldom required, and they may be taken by the pocket-sextant if the distances are not less than 300 or 400 feet, thus avoiding its parallax, and taking care to measure them in a plane but little inclined to the horizon. An angle of 90° is right at any inclination, all others are more or less too great. Railway curves, always circular, are easily described

* In both cases back angles must be used.

on a plan by drawing perpendiculars from the point proposed for their commencement and termination, supposing the meeting of the two straight lines and the distance where the curve is to begin and end is known (both being necessarily the same). We investigated a formula for that purpose in March, 1840 (see diagram and calculations in Appendix), a centre not being so easily found in the open country, where it may fall in a wood, water, &c.; nor can we hope to describe a curve of large radius on the ground itself, by other means than working around it. This method is frequently productive of difficulties in hilly or woody countries; and in towns and some other places fails altogether. It will be found generally preferable in chain-surveying to draw a rough sketch, when the quantity is small, and place the dimensions in their proper order, though some persons prefer a field-book.

BOOK II.

CHAPTER I.

INTRODUCTORY OBSERVATIONS; AND DIVISION OF MILITARY SKETCHES.

THE foregoing sheets have been entirely devoted to the practice of local surveying and sketching, which, for the most part, is expected to be very accurate. In estates, where the value of land is one of the principal objects of a survey, the content cannot be too carefully ascertained; and in filling up the triangles of a trigonometrical survey, although not quite so much attention is required, because many small things cannot be noticed in it, yet, as the best instruments, much time, and large sums of money are employed and expended, the angles and distances must receive a great share of attention; every feature that the scale will admit must be preserved, and much skill be exercised in putting the whole together, or the great sums necessarily expended upon it will be wasted, and discredit to the persons employed will be the inevitable result.

A survey of large extent absolutely requires that a base or bases should be measured with the utmost care, and that series of triangles should extend from it in every direction, so that the whole country is covered with them, and therefore the situation of every remarkable object determined with the utmost possible accuracy; then the filling in is to be done, as shown in the instructions for surveying, the triangles first laid down becoming a test of its accuracy.

It will be remembered that the roads, rivers, &c. are surveyed by bearing and distance: hence their correctness, both as to length and also their true serpentine form, when proper care has been taken: this is the method in very extensive works, as kingdoms, provinces, &c.

In smaller surveys, not exceeding ten or fifteen square miles, if we carefully survey the boundary first, and then the interior roads, occasionally intersecting some remarkable object by way of check, no base will be necessary, for the whole is a continued series of bases of various lengths; and experience shows that we may safely rely upon the theodolite and chain in such small pieces. It is when great length of roads, rivers, &c. are thus surveyed, that we need the establishment of certain points by means of triangles, in order that we may be assured of an inaccuracy when it occurs, and how to correct it, by adhering to those points; exactly upon the same principle as a mariner corrects his course, by obtaining his latitude and longitude as often as possible.

In military sketching it is somewhat different : the instruments must be portable, and therefore small, the time frequently very limited, and the sums expended upon it much smaller than on the first-mentioned surveys. Its objects, and the circumstances under which it is performed, being thus so different from the more operose surveying, the shortest, easiest, and most certain methods of practice will ever be entitled to the greatest attention.

Despatch and simplicity of execution are the great things to be aimed at in a military sketch ; and although the greatest possible accuracy may not be absolutely necessary, yet this circumstance should not become a cloak for glaring errors : for it must be remembered that these sketches are often the most authentic sources of information we possess as to the topography of distant countries ; and if they are very defective, there is only one thing in their favour, namely, that when still further reduced into geographical maps the errors are considerably diminished. The errors to which we at present allude are, first, the general outline being incorrect, because distant points have not been fixed from lines of sufficient length, or by using very imperfect instruments ; secondly, that the hills or mountains, not being drawn with a proper regard to their real form or steepness, they cannot be properly connected when the separate portions of a large work are to be blended together in one general map.

It may be convenient to separate military sketches into two divisions : first, the rapid sketch of a po-

sition in advance, or of a battle immediately after it is fought, to be sent with the despatches, or of a line of route, &c. &c. These may be done secretly with but little assistance from instruments, sometimes without any; and as they are to serve only a transient purpose, much latitude must be allowed to those who perform such service. Secondly, such sketches, or rather surveys, as may be undertaken by officers at periods of greater leisure; yet not admitting of a numerous party, with elaborate instruments being employed upon them, and also frequently requiring some degree of secrecy.

From the first division of military sketches we cannot expect much: when they have served their original purpose, their imperfect execution excludes the further use of them when any thing better can be found. It is to the second division that the most importance will always be attached: it is a collection of these that will ever be considered valuable in a military and geographical, perhaps we may say, also, a geological, point of view, and therefore the errors before mentioned should be avoided as much as possible.

The principles of military sketching cannot differ essentially from those of surveying; they both consist in determining the sides and angles of real or imaginary figures upon the surface of the earth: these are always resolvable into triangles, by means of which we lay down these figures upon paper to any required scale. But the practice differs very considerably; and it is for this reason that they are

called sketches rather than surveys, because so much of them is usually done by the eye, instead of being a continued series of angles and measured lines, as in the more elaborate surveys. The practice of drawing the hills in the field, or the drawing-room, differs only in a more hasty manner being adopted, partly for the sake of despatch, and partly because having been more hastily sketched they are probably less correct than in a survey where despatch was not so indispensable.

As, in surveying large tracts of country, large triangles must be first formed, with great care, to find the true relative situation of distant objects, and these again subdivided into smaller ones, until there is no longer any fear of the errors of mere surveying accumulating too far before they are checked by reference to those points,—so in military sketching, when a tract of country is to be drawn, containing 100 square miles or more, a similar proceeding cannot be safely dispensed with; for nothing can ensure a proper degree of accuracy but a triangulation of some kind.

It cannot, therefore, be too strongly recommended to persons employed on this service, to pay great attention to these points; and it is hoped the description we shall shortly give of the instruments generally employed, and the methods of using them, will merit the attention of those students whose future time may sometimes be partly spent in this branch of military duty. It may not be improper to mention, in this place, that it is usual to consider all military plans whatever as made up of two com-

ponent parts, one of which is called ground, and comprehends the variety of surface only: the other, called detail, embraces roads, rivers, cities, towns, villages, marshes, woods, fords, bridges, and every other minutia, the existence of which can be essential in a military point of view. We have adhered to this distinction in this place, although not in surveying, because it is the practice, in some cases, to express by certain characters the various objects constituting the details of a plan, and where no time can be spared for drawing them as they really are: while, in surveys, every house, or cluster of houses, &c. &c. will always be drawn as they happen to stand; every road, with hedge or other fence, will be shown by two lines; and when passing over a common or otherwise, and not bounded by fences, it will be shown by two dotted lines, the line always showing a defined boundary, and the dotted line one that is not defined: whereas, in military sketches, a few houses, without reference to their precise disposition, or a single spot, denotes a village; a single line will be a bye-road; a double line a post-road; a dotted line a footpath; a circle, with small teeth, a watermill; and so on, as in *Plate II.* We shall not discuss the advantage of retaining these characters, as a four-inch scale is abundantly sufficient for every important object to be drawn as it is in nature, except being, as before said, somewhat larger; but in geographical maps, or any others on a small scale, which preclude the possibility of doing without them, or in very hasty sketches, they are certainly indispensable.

Many instruments have been contrived for military sketching, each of which has some advantage peculiar to itself; but the only ones we shall mention are, the surveying compass, Sir H. Douglas's reflecting semicircle, the pocket sextant, and the plane-table, which, as we have simplified it in the foregoing description, is a capital military instrument, when used for the second division of military sketches.

To these must be added a case of leather to hold the sketches, and an ivory protractor to lay down the angles which are taken by the compass, and also the distances: these are contained in the sketching-case, with a pencil. The sketches are drawn upon paper or asses' skin: the latter has, perhaps, some advantage over paper, from its not imbibing the moisture of the atmosphere, and therefore preserving greater uniformity in the strength of the black-lead pencil: still, as no two pencils are exactly alike in texture, when one is exhausted, another may not match the work done by the first, and the difference is not greater than what arises from the unequal degree of moisture imbibed by paper in different states of the atmosphere. The skin is also very expensive: hence, when the sketch is transferred to paper, it is rubbed out to make room for another, and the original sketch is then lost altogether. Upon the whole, and considering the advantage of preserving original sketches, paper mounted upon cloth should be preferred when these instruments are used.

THE SURVEYING COMPASS.

(Plate III. fig. 1.)

This instrument has an advantage over all other compasses. It is a complete azimuth compass, contained in a box about three inches in diameter, and now generally mounted on legs, which gives it a decided advantage over those that are held in the hand: it is furnished with a plain sight, and a prismatic lens, which slides in a dovetail groove, to adjust it to distinct vision. When the wire of the plain sight is placed upon a distant object, a reflected image of the compass-divisions is likewise seen through the lens, and the wire both bisects the distant object and marks the bearing upon the limb, which is numbered from 0° to 360° . The difference between any two bearings is evidently the required angle between two objects. The construction of this instrument will be easily understood by reference to the Plate.

An angle may be read to about $15'$ by estimation, when the compass is not shaken by the wind: when that is the case, it is usual to observe the distance any division vibrates on each side of the wire, and take the mean. It is easy to see that this is only a theodolite in miniature, and that the angle cannot be depended upon for very accurate work: it is, nevertheless, a useful military instrument where extreme nicety is not required.

THE REFLECTING SEMICIRCLE.

(*Plate III. fig. 2.*)

To combine as many advantages as possible in the same instrument, Sir H. Douglas invented, some years since, a kind of sextant, by which an angle may be both taken and protracted at the same time. It is a semicircle of brass, furnished with an index mirror, and also an horizon-glass, like other reflecting instruments; and as the angle taken by these instruments is always double the real angle, he has placed the index-glass upon the extremity of a diameter, and not at the centre: by this means the observed angle is reduced to the true angle, and is protracted in the usual manner by a projecting radius of the semicircle, which moves around it as the index-glass is adjusted to the object.

This instrument is more to be depended upon than any compass; for it reads to one or two minutes, and is not subject to the inconvenience of shaking, as a compass must be when held by the hand: but the angles taken in planes oblique to the horizon require either a reduction or the use of an artifice, which will be given further on, after describing the sextant, to which it equally applies. *Fig. 2.* is a representation of the reflecting semicircle. The index error, common to all reflecting instruments, may be removed by adjustment: the angles will then be true, provided the objects are not nearer

than 100 yards. In this case, the parallax, also common to all reflecting instruments, will be sensible: it may amount to a few minutes if the object is near. In the sextant, this parallax is always read upon the arc of excess, which precedes the zero, and may be, therefore, found and subtracted from the observed angle, because it places the zero, with respect to the nearest object, beyond the true zero: but in the semicircle we are now describing it cannot be found; for the instrument is at zero when quite shut, and therefore has no arc of excess.

THE POCKET SEXTANT.

(*Plate III. fig. 3.*)

This useful little instrument differs from the common sextants in being smaller, and having its mirrors contained in a box of about the same size as the compass before mentioned: it is very correct, and has the advantage of a rack-and-pinion motion for its adjustment; nor can it be put out of order without violence, its parts being so well protected by the box. It is also furnished with a small telescope, which sometimes slides into the box: this may be removed, if the observer is satisfied with the plain sight. Being also furnished with dark glasses and the necessary adjustments, it may consequently be used for every purpose to which a reflecting instrument can be applied. Even the latitude can be found by a single observation of meridian altitude within a minute, and the longi-

tude near enough to be a valuable approximation in places but little known. By a mean of fifteen observations on the sun and several stars, the latitude has been found within $1''.1465$, when the instrument read to half minutes.

CHAP. II.

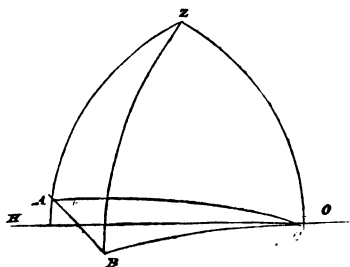
TAKING ANGLES WITH MILITARY INSTRUMENTS, AND REDUCING THEM TO HORIZONTAL PLANE INSTRUMENTALLY.

THE angle between two objects is taken by either of the reflecting instruments in the following manner:— The observer looks at the left-hand object through the telescope, or plain sight, and by the rack-work of the sextant, or merely separating the projecting index of the reflecting semicircle from the part it touches when shut up. He moves the index until the required object is brought by reflection from the index-glass, upon or in contact with that seen by direct vision, and the angle is read off upon the limb, or protracted, as the case may be, or either instrument be used. If it be an angle of elevation, the elevated object is brought down to the horizon, or to its image seen in a basin of water or mercury, and in the latter case half the observed angle is the true elevation, because the image seen in the basin is as much below the horizon as the real one is above it.

The plane-table has been before described. Now, either of these instruments will enable us to produce a military plan. As far as mere angles are concerned, the semicircle and sextant are evidently most correct; but both require that the angles taken in

oblique planes should be reduced to true horizontal angles; and this cannot be done without knowing the angular elevations and depressions, which when small are not appreciable by reflecting instruments: thus, we must have recourse to artifice to accomplish what falls without the reach of our instrument by the legitimate means. We shall premise that the compass cannot take an angle nearer than about $15'$, independent of its errors of centering, and others arising out of its construction; but we shall show how, in most cases, the errors of the other instruments which happen when objects are in planes oblique to the horizon may be reduced far below that quantity, and often made to vanish entirely.

We shall suppose (see annexed diagram) that A is an object 4° above the horizon, B is 2° below it, and C also 1° below; the angular distances from one object to another, in the oblique planes, being $\angle C = 110^\circ$ and $\angle B = 20^\circ$, as in the figure following:—



In this diagram, let the line HO represent the horizon; ZA , ZB , ZC , the distances of the objects from the observer's zenith. Now in $\angle ZB$ and $\angle ZC$

there will be given the three sides to find the angles at the zenith, which are measured upon the horizon, and are the true horizontal angles. These angles being calculated, will show that if, instead of taking ΔB as given by the instrument, we take the difference between ΔC and ΔB , we shall get the horizontal angle very nearly : —

	Deg.	Min.	Sec.		Deg.	Min.	Sec.
The true horizontal angle between A and c . .	109	58	46.8				
The angular distance measured by sextant . .	110	0	0				
Difference too much by instrument . .	0	1	13.2				
The true horizontal angle between A and B . .	19	5	25.6				
The angular distance measured by sextant . .	20	0	0				
Difference too much by instrument . .	0	54	34.4				
The true horizontal angle between B and c . .	90	53	21.2				
The angular dist., as would be taken by sext. .	90	51	30				
N.B. By calculation it is $90^{\circ} 51' 30''.7$, which would read $30''$.							
Difference too little by instrument . .	0	1	51.2				
Hence, Δc by in-	Deg.	Min.	Sec.	True horizont.	Deg.	Min.	Sec.
strument	110	0	0	angle by cal-			
				culation	109	58	46.8
B c by ditto ...	90	51	30	Ditto, ditto.....	90	53	21.2
ΔB by ditto ...	19	8	30	Ditto, ditto.....	19	5	25.6
ΔB by calc. ...	19	5	25.6				
Difference between calculation and measurement...	0	3	4.4				

From the preceding statement, we learn that in really using the angles taken in the oblique planes, there would be an error of $54' 34''.4$ in the angular distance between A and B, which, by taking the difference between ΔC and ΔB , as actually measured, we reduce to $3' 4''.4$. We shall now take another case, where A is 1° above, B 1° below, and c 1°

above the horizon; AB, by measurement, 32° , and AC 97° :—

	Deg.	Min.	Sec.
The true horizontal angle between A and c	97	1	11
The angular distance measured by sextant	97	0	0
Difference too little by sextant	0	1	11
The true horizontal angle between A and B	31	56	20.6
The angular distance measured by sextant	32	0	0
Difference too much by sextant	0	3	39.4
The true horizontal angle between B and c	65	4	50.4
The angular dist., as would be measured by sext.	65	4	0
N.B. By calculation $65^\circ 4' 11''.4$, but the $11''.4$ could not be read on the limb.			
Difference too little by sextant	0	0	50.4

	Deg.	Min.	Sec.		Deg.	Min.	Sec.
A c by instrument ...	97	0	0	The true horizont.			
				angle by calcula-			
				tion.....	97	1	11
B c by ditto	65	4	0	Ditto, ditto.....	65	4	50.4
A B by ditto	31	56	0	Ditto, ditto.....	31	56	20.6
A B by calculation ...	31	56	20.6				
Difference between							
calculation and							
measurement.....	0	0	20.6				

In this last supposition, which is much nearer the usual practice, the first being an extreme case, the angular distance between A and B, which would err $3' 39''.4$ from the truth, is obtained by taking the difference between it and AC, within $20''.6$: the error may, therefore, be considered as evanescent; for no instrument in common use, for laying down angles, can do it nearer than $1'$. In the other angles the difference is also very trifling. Hence, if two objects lie in a plane very oblique to the horizon, we must take an object far to the *right* of them, and measure

the angle between that and each of them, and the difference between these two angles will be very nearly the true horizontal angle required. In taking angles by reflexion, the brightest object is always made the reflected one; and thus, when the brightest is to the left hand, we are sometimes obliged to invert the instrument, or hold the face downwards: in this case an inversion of the rule just now given must take place, and an object far to the *left hand* must be taken.

Now an error of 3', in a distance of 12 inches, on paper, would only become sensible in very acute intersections, which should always be avoided as much as possible, or rejected until we can confirm them by others less acute; but 12 inches usually represent from three to six miles upon military plans, so that by avoiding acute angles, and selecting such objects as appear nearly horizontal, and when we cannot do so, by taking the difference between two angles as the true one, we may make the reflecting instrument perform its office without sensible error in almost every case that can possibly occur; indeed, triangles of six or eight miles to the side have been interposed between those of a large triangular survey with great success, by the means just pointed out, but the sides were always calculated, and the triangles laid down by beam compasses.

CHAP. III.

FIRST DIVISION OF MILITARY SKETCHES.

WE now begin with the first division of military sketches, properly so called, as the second may be made to vie with surveys, if carefully conducted; and a specimen, with its description, will be introduced in illustration. A sketching case, pencil, and scale, alone were used upon this small sketch, which might be continued indefinitely, by working on each side in succession, as the best means of dividing its errors, if that were deemed of sufficient consequence.*

One of the most essential things to be acquired, is that of judging distances with accuracy. Upon this every thing depends in a hasty sketch, where instruments are sparingly used or excluded altogether. It is commonly acquired thus: the value of the pace is previously ascertained upon a measured distance — a long distance is best, as the mean value of a pace can be established upon it with greater exactness than on a small one; a distance is then judged, and afterwards paced; the difference is noted, and the practice

* Commons are now constantly sketched by dividing them into triangles, pacing each side and the offsets, noting them on a rough sketch made at the time, while the disposition of encampments on them is made by the regulation plans from the quarter-master-general's department.

continued until the student is competent to trust himself in judging distances of considerable length. He is now capable of sketching by this method, transferring the principles and artifices of surveying into it as completely as the exclusion or absolute want of instruments will admit; that is to say, judging, or, if he pleases, pacing his distances, and operating exactly as he would do were they measured, and his angles also.

The following description (*Plate IV. fig. 1.*) of an actual sketch will place this subject in a proper point of view. At *a* the sketch was commenced in the direction of the canal produced, and the canal drawn of an indefinite length; the wharf, pond, and houses at *g*; the cottages and other objects about *a* were drawn entirely by judging their distances and positions with respect to the line *ab*, which was paced, and the ground drawn at the proper intervals, judging the distance right and left, exactly as in surveying and using offsets: then the line *cb* was drawn as nearly true to the proper angle as could be done by holding the sketching-case, as a plane-table, with the line *ab* in its proper position: *cb* was paced, and the ground put in as upon the former line; then returning to *b*, a direction to *h*, and afterwards to *c*, gave by the same means the ground in that direction; and upon arriving at *c*, the road to the bridge at *d* was drawn as the other lines had been, and the direction of the canal both ways joining it to the wharf, pond, &c., producing it indefinitely in the other direction, because it runs straight far out of the

limits of the sketch. The direction of the road *d e*, and from *e* to the gravel-pit, were drawn as near as possible, by imitating the use of the plane-table, always putting in the objects right and left at assumed or paced intervals, and occasionally quitting the line paced upon, in order to see the ground better. Lastly, the direction of the canal from *f* to *g* being already given, its length was judged, which determined the place of the bridge; and the road from *f* to the gravel-pit being judged at a certain length as the other distances had been, this assumed length, with its direction, as ascertained from the bridge, afforded some criterion of the accuracy of the sketch, analogous to what is called closing, in regular surveys. But we do not mean to say that closing exactly can be expected in a work of this kind, more especially when a person may have gone round a considerable circuit; for in works of this hasty nature it is not necessary, the greatest latitude being allowed in such operations. Our sketch has been compared with surveys and better sketches done by the help of instruments afterwards, and the resemblance is quite sufficient for the purposes to which such sketches are applied: had it been extended into the adjoining enclosed country, the distances and directions of such farm-houses, points of hills, &c. &c. as presented themselves, would have been judged exactly in the same manner, and the roads drawn from one place to another as near as the eye could estimate while going over the hills themselves; and thus a rough document may be produced with very

little trouble, constituting what is called reconnoitring or sketching a position in advance. But where the features are large and commanding, it is evident that a skilful person will much shorten his labour by sketching ground at a still greater distance from the places where he stands than we have done; and it must be observed that an accurate acquaintance with the varieties of hills, from long practice, will be eminently serviceable. In this manner we may also form what is called a column of route, or a line of road of many miles in length, judging the bends of the roads and the distances*, remarking every object, right and left, within perhaps half a mile, with the nature of the roads, bridges, and every thing of military consequence, in a suitable report annexed: or we may pass from height to height, if the road lies among hills, and draw the road as we go, similar to what has been before said, and will again be mentioned, when we show how to make more elaborate sketches, freely using such instruments as we can command.

Before quitting this subject, it may be right to inform the student, that in this rapid sketching Sir

* It is now common to produce these by laying the sketching case on any support that offers, or on the ground, and working by the back angle, as with the plane-table, which prepares learners for it effectually, and gives the bends almost correctly, always drawing the forward angle backwards when at the end, and forwards at the beginning of the paper, which renders the junction of many such sketches easy and nearly certain. Lines of road have been thus carried on 12 miles or more very frequently.

H. Douglas's semicircle is, perhaps, the best instrument to be used, if an instrument be allowed, because it will determine the places of houses, &c. with great despatch and correctness, thus leaving very little to be done by judging or pacing distances: for a large piece may be rapidly sketched, even on horseback, if a single distance is first assumed as a base, and from it several objects fixed, which leaves nothing more to be done than filling in the sketch by the eye; but the circumstances under which military operations are performed are so extremely varied, that an officer must be left to judge for himself upon the necessity for using or rejecting instruments according to them.

When a person has been regularly taught to make correct plans, by any of the methods we have given, or shall give, he will find that barely riding over ground, and examining it with the eye of a military man, will be sufficient for him to describe it roughly upon paper, and that such a sketch as he can produce by this means, especially if aided by a few angles and notes that take little time to obtain, will communicate a very competent knowledge of the ground to any other officer, for whose use it may be required. It is a practice with many persons to obtain, if possible, provincial maps, and lay down the ground upon enlarged outlines from them; but unless these maps are good, we doubt whether much is gained by this proceeding: for equal error will arise by hastily enlarging a bad map, as in trusting to the eye, occa-

sionally corrected by a few angles; nor can such maps always be had: indeed, any plan enlarged, unless it be of triangles or buildings, which have the dimensions actually written in their proper place, will always be a failure.

CHAP. IV.

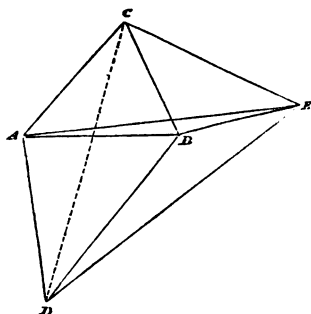
SECOND DIVISION OF MILITARY SKETCHES.

THESE must be conducted in a manner exactly the reverse of the theodolite survey. It will be in vain to expect that pacing and taking bearings by a compass, through a winding road of great extent, can possibly be true: it may be barely trusted to for short distances and in straight roads, but not otherwise, for the angles are often uncertain, and so are the paced distances in many cases: we must, therefore, devise a method which will obviate these objections; and while it ensures general accuracy, throw all the errors upon those parts which are of the least importance.*

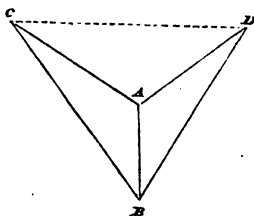
We begin by measuring, or, at least, pacing a base as long as possible: if paced, it should be gone over two or three times, and the mean of them taken for the true length. There are often great difficulties in selecting a spot favourable for this purpose: it most commonly happens that, when a spot has been found, it lies so inconveniently as to make the intersection of distant objects from each end too acute to be trusted to; this inconvenience may be obviated in the

* See note at p. 124.

following manner.— Suppose the base AB (annexed diagram) to have been measured, and that E is an

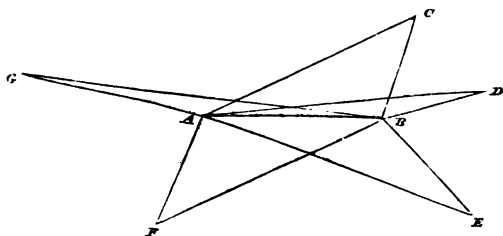


object so conspicuous as to be of consequence in the sketch. The acute intersection AEB cannot be depended upon; but if two objects C and D are selected and fixed with great care from A and B , and then checked by taking the angles ADC and CDB , or ACD and DCB , their places may be considered as certain; and from CD , as an enlarged base, the place of E may be more accurately determined. This has been done on large trigonometrical surveys; and when of so much importance, four values of CD can be had by calculation, the mean of which will be very exact; but in military sketches it is enough to determine the places of C and D by three intersections passing through the same point. Other methods may also be practised for securing the same advantage: thus CD (annexed diagram)



may be determined from A and B, and being proved from C or D, will present an enlarged base more conveniently situated to fix other points, than the original base AB; indeed three bases will then be formed, BD, DC and CB. Many other varieties of this method will readily occur to the observer, according to the nature of the ground, and he will of course decide for himself.

Now, from points thus fixed with much care, other objects may be determined, always, if possible, in the larger triangles, having three lines passing through the same point, and the work will be more correct than when we determine too many from the original base, as in the annexed diagram, a practice much to be avoided; for the errors occasioned by so many acute



intersections at the commencement, if they are not checked properly from one another, or others better situated for that purpose, will infallibly derange the whole work. It must be laid down as a principle that the most conspicuous objects are to be most correctly placed, and the minor points derived from them; by this means we command the greatest possible accuracy, and throw all the errors upon those parts where they can be propagated no farther.

It is a good method to take as many angles as possible at each station, and make our selection of the greater triangles afterwards, unless, from a previous knowledge of the country, that has been already arranged; and it will generally happen that we can see the interior of a triangle from at least two of the sides which enclose it; if not, the points therein must be fixed from any other side of any adjoining triangle where they can be seen, as each side of a great triangle becomes a base for fixing interior points.

The triangles are generally laid down by a protractor, in military sketches; but as we suppose, in this division, that time and instruments are allowed, and as we can produce a sketch little short of a real survey in point of accuracy, we have no hesitation in recommending that those triangles, which may have sides of one or two feet in length upon paper, or from three to six miles in length upon the four-inch scale, should be calculated and laid down by the length of the sides themselves. This is our own practice, because we know that when these lines exceed eighteen inches, let the scale be what it may, they will become very liable to error if continued too far by a protractor; and hence, if only a few of the greater triangles are calculated and so laid down, we are assured of their accuracy, and also of the interior points when laid down by a protractor.

The method about to be described depends so much upon the correctness of the triangles, that they must be managed with the most scrupulous attention.

The instruments chosen for finishing the sketch after the triangulation is finished, will be either the plane-table, or the sketching-case, and skins, aided by the surveying compass; for the triangles themselves may be formed by the sextant or theodolite at the option of the person employed.

Of the first-mentioned instrument, which is far the best for expedition and accuracy, we need say but little here, as it would be almost a repetition of what has been before taught under the article Surveying. What remains is common to it and the sketching-case, only with the difference arising from the method of mere sketching, as it respects the plane-table, and laying down the work when we use the sketching-case and skins.

Having determined as many points as may be deemed necessary, some of which should fall without the limits of the sketch, and will be productive of great advantage while sketching near its boundaries, as well as when joining it to others, — we must dot them down from the paper they have been constructed upon, to the skin we propose to use, and mark their names. Now we go to one of those points and take the bearing of some other, the more distant the better; and connecting the two points in question, we, by means of this bearing, lay down a line representing the magnetic meridian of that particular compass employed, for they generally differ a little in different instruments, as before noted, and also many others at about an inch apart: we are now in condition to begin our sketch in the manner following.

Before we quit the place, if any ground, road, or other object necessary to be noticed, is either there or near it, we draw it on the spot, and then, always keeping upon the highest ground, find our place by taking the bearing of any two or three objects forming the points of our triangles, and laying down those bearings from the objects towards ourselves. We thus ascertain our place correctly if these lines meet, as they will when good instruments and care are used. This practice is common to this method, and that by the plane-table; but the latter instrument does it more quickly and more correctly, being less liable to error than laying down bearings by a protractor when taken by a compass, thus including the error of both these instruments. The place being found, we proceed as before to draw the contour of the ground, &c.; and supposing a road, house, stream, &c. to be near, we take a bearing to some point upon it, and pacing the distance, examine the object and mark this point, sometimes taking also a bearing both ways from that place, in the case of streams or roads, and drawing either as far as we can trust ourselves while upon the spot. Again finding our place upon some adjoining feature or height, and doing the same, we join it to what was last done by the eye, occasionally assisted by the instruments; and by continually keeping upon the high ground, where we can see fixed points within a mile or two, we find our places; and from the places thus determined, other points in roads, &c. are found, and the intermediate bends drawn by the eye on the

spot; thus ensuring general, and also particular accuracy if we please, by a continual subdivision of errors: we also avoid the fatigue and inaccuracy of pacing over unfavourable ground, and save much time by not actually going through the roads as in surveying. But many other things are done as we go on: for example, at the places thus constantly found, we intersect houses and other objects, but particularly houses, because they are always upon or near a road, and, therefore, determine a point of some value in drawing in those roads; and in passing by woods or commons, we always do as much of their boundaries as we can see from that particular spot; and also, if in passing, we are in line with any boundary or road of considerable length, and nearly straight, although at a mile or more distance, we find our place, and take a bearing in the direction of its length, and thus prepare the distant parts of our sketch before we actually come to work upon them. The difference between sketching and surveying, which was said to be exactly reversed, will now be evident: for, in the former, we derive the roads, &c. from the hills; while, in the latter, the hills are derived from the roads, &c.* The military officer has more to do with the hills than the roads: if the latter are not perfectly true as to their flexure, so that the former are well connected and expressed, the plan will lose

* Many persons prefer obtaining the roads, &c. by bearing and paced distances, checking themselves continually by fixed points, and thus produce sketches little short of surveys in accuracy; it has been found best where the roads are very numerous and a plane-table used.

nothing in the estimation of the general, who should nevertheless know exactly the state of the roads with respect to breadth and goodness, and the streams as to practicability in fording, their bridges, mills, &c. ; but the ground is indispensable, and so are the general dimensions of wastes or places fit for encampments, as well as woods and the nature of their trees, whether timber, of what kind, what underwood, and, in enclosed countries, the nature of the timber in the hedges, &c. ; but these things more properly belong to the written reports that should in all cases accompany plans made for military or political purposes.

We must now observe that there will sometimes occur cases, in which, from a paucity of points, or when from being entangled in small ravines or otherwise, many will disappear, we are obliged to have recourse to pacing, no other method being practicable : but this does not invalidate the system ; for filling in small intervening spaces becomes at length so familiar to practised persons, as to present no great difficulty in any case whatever. To avoid this inconvenience, we always sketch around the difficult places, and by invariably diminishing the space, circumstances will always enable us to fill it in by mere reconnoitring ; and besides, we constantly determine, by intersection, the places of many objects in the parts that appear likely to produce this difficulty, and thus often avoid it altogether.

There is also another case, namely, a flat enclosed country, where we must pace and take bearings,

unless we can regularly survey it, for it is obviously impossible to fix points, or find places, if they were fixed; but this will not often happen; and with respect to extensive forests, it is well known and universally acknowledged, that there is no other way of sketching a country so circumstanced, than by examining its parts at such convenient distances as may present themselves, and sketching the mountains by studying the nature of the ground, and its analogy with surrounding features, or those we have seen similar in other countries.

It is evident that much must be done by the eye alone in military sketching. The greatest advantage is to be derived from the accurate use of that organ; and whenever it saves an instrumental operation we must by no means neglect it.

The nature of roads may, in great measure, be inferred from the surrounding soil; for, whatever may be its component parts, they will influence the state of the roads materially, which, for the most part, are composed of materials near at hand: this is one advantage of knowing, from the configuration of hills and mountains, their probable, we may almost say certain, composition. But that nothing may be left to conjecture in a case of such importance as the march and subsistence of armies, this subject should always be included in the written report.

When a plane-table is used for these sketches, the plan of operation differs in nothing but in the method of laying down the bearings, as before described under the article Surveying, the points being dotted

down on paper mounted on the table; and the degree at which the needle stands when two of the points are in line with the original points upon the ground, being marked, as 3° S. W., or whatever it may be, will always serve to place it parallel to its first position, and then the proceeding is exactly as before detailed. When one sheet is filled, we dot down the border of it, which it is proposed to extend, and a few more points in that direction, two of which should be common to both sheets, and then determining the position of the needle, proceed as before: the very same thing as must also be done in using the skins. If a boundary between two adjoining sketches is gone over by two parties, and if both work indiscriminately from each other's points, the plans must join properly.

We may now observe, that the difference between the accuracy of the plane-table and the other instruments consists chiefly in this:—The plane-table is always adjusted to the same degree or part of a degree, and therefore, whether the compass-card is well divided and centered, or not, we are sure of being right, if the table is very nearly horizontal; also the angle or bearing being drawn by the side of the rule, is only affected by the small parallax of the instrument, which we have before shown to be generally insensible, and wholly disappears if the place where the bearing is to be taken from is placed over the station. But the card of the surveying compass may be badly centered; therefore, as we take angles on every part of its limb, these angles or bearings must

constantly vary a little from the truth; but a still worse source of error remains, connected with the ivory protractor, and the use of it in the field, — this instrument being seldom accurately divided. The skin not laying quite flat, and the unsteadiness of the case when held on the knee, or on the ground, also present great difficulties to laying down lines with exactness: hence some officers are incumbered with a camp-stool to sit upon, or to support their case, when performing this operation; and it is obvious how much time is thus lost, and with what uncertainty the work is performed, especially if E. and W. lines are used. If N. and S. lines were used, and the protractor according to the old method, rather greater accuracy, and certainly more expedition would be attained. When E. and W. lines are employed, short transverse lines are drawn across the ivory protractor, and the angle is protracted by placing the centre upon the angular point, adjusting it by any E. and W. line that may happen to coincide with one of them; then dotting off the required degree to turn it to the two points, and rule the bearing which is thus done at twice. If meridians are used, nothing more is necessary than to place the ivory protractor upon any convenient one, and adjust the centre and required degree upon that meridian, to slide it forward or backward until its interior edge passes through the angular point, drawing the required line to whatever length may be necessary; the subjoined diagram will explain this.

The bearing and distance fg is required to be laid

off from f ; place the centre of the protractor upon h i , and the degree upon this meridian, then sliding until the back edge cuts f , rule the line $f g$, and lay down the distance; another bearing from g is done in exactly the same manner, but it must be noted in which direction we are proceeding.

Plate IV. fig. 2. is a specimen of sketching by this method, with a plane-table, in a country partly open and partly enclosed with thick hedges and large timber trees, from the points laid down from sextant observations as before detailed: there have not been one thousand paces employed on the whole of it, and in a continuation of the same sketch much was done without a single pace. The objects employed for determining the places were many of them one or two miles distant; and although the compass-needle does not exceed two inches in length, yet no difficulty occurred in finding those places correctly, nor in seeing the points while on the high ground, as should always be particularly attended to.

Connected with this system of sketching, and indeed every triangulation whatever, we must observe, that great difficulties in fixing points will occur without management: some practical instructions will now be given how to avoid them.

The most elevated points can always be seen almost every where, but we want many more amongst the lower chains of hills, and consequently in valleys of difficult access to the sight.

Now we find that while determining points trigonometrically upon the inferior *ridges*, we can also

see up the opposite ravines, or the opposite side of an intervening ravine ; and when upon that opposite side, we can easily do so by the hills we have left, and thus are able to determine numerous objects, by which we find our place, when sketching from the points on the opposite side of a valley : and this illustrates a position we have before advanced, namely, that it is better always to fix points out of the boundary of our sketch. To this may also be added, that it is frequently found impossible to measure a base, or form triangles upon the piece of ground to be sketched ; and, therefore, both must be done upon some adjoining part more favourable to these operations : or else we must survey the whole by pacing and taking bearings without any other checks than can be derived from frequent intersections of some one or more remarkable objects within or without the boundaries of our sketch, according to circumstances.

CHAP. V.

METHOD OF SURVEYING IRREGULAR INTRENCHMENTS, ETC.
BY THE SEXTANT.

It may sometimes happen, that to know the form of an irregular line of wall or intrenchment is an object of some importance, and the approach may be difficult, or, perhaps, impossible: by the following method it can be sketched very accurately at such a distance as to secure the officer from unnecessary danger, and, perhaps, in some cases, from observation. (*Plate I. fig. 1.*) Let $ABCDEF$ be an irregular line, with a marsh before it, and not to be approached nearer than the line KLM .

Now, if we set out from K , first taking the angles AKL , BKL , and pacing towards an object in the line KL , observe at what distance from K any of the lines of the fortification produced would reach the point where we stand, and take the angle subtended between them and the line whereon we walk, doing this as often as we are in those lines produced; and having arrived at L , suppose that we are obliged from circumstances, or deem it advisable to change our route and advance towards M , doing the same as before, all through the line, we shall then obtain a plan far more correct than we could do by mere intersection of the different angles of the fortification, because, in that case, acute intersections would often

give a false length to those lines which were short ; and in this particular instance of sketching, we have not always the means of avoiding such acute intersections. A reflecting instrument is the best for this purpose, and Sir Howard Douglas's is admirably adapted to it, because the lines may be laid down upon the spot, and, indeed, many other sketches may be made in a similar manner by the help of this instrument.

Coasts, as well as rivers, are sketched exactly as the roads, by finding certain places upon their banks, and connecting them, the principle extending to all possible cases ; but the intervals should not be so long as to make it difficult to see the bends lying between any two of these points.

Streams are frequently drawn at the same time as the features between which they run, except in enclosed countries, where they are often diverted from their original course and conducted along the sides of roads ; if such nicety is required, they should, therefore, be noticed while drawing those roads, as in regular surveys.

Fields are seldom delineated exactly upon surveys, when the scale is less than eight inches to a mile ; much less upon military sketches, otherwise than by making them according to fancy, wherever they may be, instead of regularly surveying their boundaries, as upon more particular plans, where the scale is as large as four or five chains to one inch.

Buildings are also drawn with less attention as to number and position, because they are only sketched

by the eye, and generally in haste: yet large buildings or towns should certainly claim some share of attention whenever time can be spared; and fortifications are first laid down on a large scale, and then reduced into the plan, if it belongs to the second division.

Remarkable trees, towers, or other objects situated on the tops of hills, should never be omitted: they serve as landmarks, and in countries not well known, and many other cases, are of great importance to armies on their marches.

It often happens, as the sheets of the sketch get filled, that great *facilities* arise from that circumstance alone; for many objects will be found to lie in a line passing through two points already ascertained, or in some line produced; a single bearing will in that case determine our place. Or we may find some other pair of objects or line produced that will do as well, by intersecting the first line supposed, and then we sketch around us as usual. Thus many little shifts, to save time and the use of instruments, will frequently suggest themselves to the person who has had much practice, and they should never be neglected.

Very excellent plans of the second class have been produced by triangulating with a sextant, and, having determined numerous places, filling in the ground entirely by the eye. Thus it must frequently happen on foreign service, for an officer cannot always procure the instruments he prefers. It is evident that in this method of sketching some substitute for

a plane-table is used, but it is no more than barely holding the sketch parallel to the original figure of the ground, as near as may be, and judging the bearings of points interior to the triangles. In mountainous countries, this kind of sketching is capable of great despatch; but in hills of smaller features, and in countries comparatively flat, and much enclosed, we must have recourse to other instruments, of which the simple plane-table, with a very light mounting, is clearly the best.

CHAP. VI.

NECESSITY OF ASCERTAINING LATITUDE AND LONGITUDE APPROXIMATIVELY, UPON FOREIGN SKETCHES OF THE SECOND DIVISION; AND METHODS OF DOING SO BY MILITARY INSTRUMENTS.

EVERY military sketch of the second class will acquire additional interest, if the absolute heights of the principal hills or mountains be ascertained, if possible, above the mean level of the nearest river, lake, or the sea*; in the latter case, it should be stated whether at spring or neap tides. All circumstances considered, it is probable that the most expeditious, and perhaps the most correct way of doing so, is by the use of the mountain barometer. Of these instruments, Sir H. Englefield's is the most portable, and least liable to get out of order, being about the size of a common walking-stick; it requires no logarithmic tables, every thing being done by that attached to it, and a trifling calculation.

The latitude and longitude of some remarkable place may likewise be ascertained, as we have said before, with the small pocket sextant; and the true

* The sea level is very different on different parts of a coast. The Trinity Board have fixed a mark called Trinity datum, at the average high water at spring tides, a little above low water mark at Sheerness. In Walker's Geological Map the elevation of the canals are all calculated from the low water mark at Liverpool.

meridian, also, by the same instrument, with the variation of the magnetic needle, when combined with the surveying-compass, if there is an observer to each instrument. The following method will serve when the sextant and compass are used, and also with the theodolite; but we shall afterwards show how it may be done with the sextant alone.

The reader will easily understand the nature of this operation, which was adverted to in a preceding part of the work, upon the following principle:—If we know the exact azimuth of the sun or a star, at any given moment, and if we also take the difference between that and the azimuth or bearing of any terrestrial object, we can thence lay down a true meridian line; for the difference between the two true azimuths and two magnetic azimuths of the objects is exactly the same; and, therefore, by a simple addition or subtraction, the direction of the true meridian is easily obtained.

This supposes the latitude of the place of observation to be known, and hence it becomes necessary to show how this element may be found by the instruments we have mentioned in Military Sketching. We shall show how we have done it with the small pocket sextant, and then proceed to the other operation, of laying down a true meridian, and finding the variation.

Take the altitude of the sun, or a star, when on the southern meridian in the northern hemisphere, or the northern meridian when in the southern hemisphere, — by bringing its image down to that

seen by reflection in a basin of water or mercury, half the measured angle will be the altitude required. Several altitudes should be taken just before and after the meridian transit, as quick as possible, and the greatest will be that nearest the truth ; for we do not yet know the exact point of greatest altitude, because the true meridian is supposed to remain unknown. If the sun has been employed, the refraction must be applied to the observed limb, and afterwards the semi-diameter added or subtracted, according as the lower or upper limbs were observed, the parallax in altitude being nearly insensible ; but if a star, then the refraction alone will be necessary.

Now add the north polar distance to the corrected altitude, and subtract the sum from 180° ; the remainder is the latitude. This is a convenient rule, as the north polar distance of stars is always given in the Nautical Almanac ; but for the southern hemisphere, we must use the south polar distance, or the supplement of the north polar distance. The polar distance of the sun will be obtained by subtracting his declination from 90 , when of the same name as the polar distance, otherwise adding it.

The following is a specimen of the calculation for a star, followed by one for the sun : —

May 16. 1822.

Greatest meridian altitude of Spica Virginis					° ' "
observed	-	-	-	-	2)56 55 30
					<hr/> 28 27 45
Corrected refraction	-	-	-	-	— 1 43.8
					<hr/> 28 26 1.2
North polar distance	-	-	-	+	100 14 1.5
					<hr/> 128 40 2.7
Latitude	-	-	-	-	51 19 57.3
					<hr/>

January 19. 1823.

Greatest altitude of ☉'s upper limb observed					2)37 5 45
					<hr/> 18 32 52.5
Corrected refraction	-	-	-	-	— 2 53.6
					<hr/> 18 29 58.9
Semi-diameter, &c.	-	-	-	-	— 16 22.1
					<hr/> 18 13 36.8
North polar distance	-	-	-	+	110 26 37
					<hr/> 128 40 13.8
Latitude	-	-	-	-	51 19 46.2
					<hr/>

In these calculations, the sextant reads to 30", and the necessary modifications by barometer and thermometer upon the mean refraction, as well as the parallax in altitude, were noticed, which many persons would omit, as the instrument reads only half-minutes, and has no repeating properties to attenuate its errors; besides, we are only aiming at such an approximation as military instruments will give; and for that reason, shall not enter into the

niceties required in geodesic operations of the first eminence, as being in some measure unnecessary either in surveys not exceeding 1000 square miles, or in any military sketches whatever.

Let us now take the mean of these two latitudes, and compare it with that obtained by interpolation from the Great Survey of England, &c. : —

					°	'	"
Lat. by Spica	-	-	-	-	51	19	57.3
Lat. by ☉	-	-	-	-	51	19	46.2
					<hr/>		
					2)103.5		
					<hr/>		
Mean	-	-	-	-	51	19	51.75
True Lat. obtained as above				-	51	20	5.6
					<hr/>		
Difference	-	-			13.85		

Many more observations have been made, rarely differing a minute from each other; and, hence, we see that the instrument will do a great deal more than might be expected. The index error has been removed by adjustment, and there has been no perceptible alteration in 12 years, probably from the mirrors not being within the reach of derangement, and the small expansion of the adjusting screw not being sensible even in the hottest weather.

We now proceed to find a true meridian from the place, the latitude of which we have found, from actual observation.

α Aquilæ.

July 2. 1823.

Alt. observed	-	25°	12'	0''	az.	135°	6'	0''	} S.E.
And	-	26	18	0		136	45	0	

And azimuth or bearing of a distant tree, 204° 0', S. W.

Z *	64	50	3	-	-	ar. co. sin.	0.0433126
Z P	38	39	54	-	-	ar. co. sin.	0.2042828
P *	81	35	23			sin.	9.6674538
	2)185	5	20			sin.	9.9072922
	92	32	40				2)19.8223414
- Z *	64	50	3				
	27	42	37			Sin. $\frac{1}{2}$ Z	} = 9.9111707
$\frac{1}{2}$ sum-Z P	38	39	54			54° 35' 23''	
	53	52	46			2	
						109 10 46 = Z.	

The magnetic bearings of the two objects will differ exactly as much as the true ones; the difference between the former is 204° 0'—135° 6', or 68° 54'; but the true bearing of the star was only 109° 10' 46'', as found by a calculation depending upon its altitude: and as the readings all run the same way, we have only to add the difference of the magnetic bearings to the true azimuth of the star, and the real bearing of the tree will be known: thus, 109° 10' 46'' + 68° 54' will be 178° 4' 46'', counted round from the north eastward towards the south; or, more properly, the supplement 1° 55' 14'' is the true azimuth from the south-eastward, and, therefore, a line making this angle, with one joining the two stations, will be the true meridian sought.

But this same set of observations will also determine the variation of the magnetic needle; for the difference between the calculated and observed azi-

muths will be the variation of the needle, subject to the following distinction of cases, viz. when the object is eastward of the true meridian, if the magnetic azimuth exceeds the true azimuth, the variation of the north end of the needle is to the eastward; if it is less, the variation is west; and when the object is to the westward of the true meridian, if the magnetic azimuth exceeds the true azimuth, the variation is westward; but if it is less, then the variation is eastward.

In the present instance, the calculated

					°	'	"
Azimuth is	-	-	-	-	109	10	46
The Magnetic Azimuth	-	-	-	-	135	6	0
The difference	-	-	-	-	25	55	14

And the variation is westward, because the magnetic azimuth is least, — if we use the supplement of these two numbers as we ought to do, because we should count the azimuth from north or south only as far as the east and west points, or 90° ; but we have here taken the angle at the zenith, as given always by calculation from the north quite round to the south, and the difference of the supplements will be the same as that of the two numbers themselves, thus: —

					°	'	"
Calculated Az.	-	-	-	-	70	49	14
Magnetic Az.	-	-	-	-	44	54	0
					25	55	14

Variations as before, westward, because }
the magnetic azimuth is least.

The other observation gives the variation $25^{\circ} 54' 46''$, and both are too much ; for the compass-needle was so small, that the eccentricity of its pivot produced a very considerable error upon its results : this does not, however, affect the determination of the true north line, nor the method of finding the variation.

As in all other cases, the mean of many results is best. If more accuracy is required, we should recommend that several azimuths be taken both of the celestial and terrestrial object, upon different parts of the limb of a theodolite, and both eastward and westward of the meridian, thus avoiding the errors of division, and that arising from the uncertain figure of the earth, which affects the altitudes considerably. Nor should the error of collimation belonging to the telescope be forgotten. By inverting its position, and also by reading the altitudes with the divisions facing east, and then west, coupling the observations in pairs, we get rid of that error, and possess the additional advantage of reading the altitudes upon two different parts of the vertical limb : thus one altitude may be calculated face east, and the next immediately following face west, the mean of which should be taken ; then some star on the other side of the meridian, at nearly the same altitude, in the same manner. These are niceties more than sufficient for military plans of the second division, when they do not exceed 1000 or 2000 square miles.

All the operations necessary may be made in an hour or two ; whereas, the same thing being done by equal altitudes on both sides of the meridian, although

they do not require the latitude to be known, will demand four or six hours' attention to complete, and frequently expose the observer to disappointment by the intervention of unfavourable weather, or perhaps the night air of an unwholesome climate.

We have taken the foregoing example from one in which the true latitude had been used, because $13''.85$ will make but fourteen seconds difference in the result.

If no theodolite is to be procured, the altitude and azimuth of the objects must be taken by the sextant and a compass at the same instant: the other proceedings differ in nothing but halving the angle of altitude, because of its being taken by reflection in a basin of water or mercury, before we apply the refraction.

By the following method, we may find the true azimuth of any object, and thence lay down the true north line upon a plan by means of a sextant alone. At some known station suspend a fine thread a few feet from the observer, and place the artificial horizon in such a position that the thread may bisect a distant station, and also be seen by reflection. Now, when the sun has nearly arrived at that vertical, bring its image down to the horizon, and keep the images in contact until the preceding limb of the sun touches the reflected image of the thread, at the same instant the thread bisects the distant station: note this altitude, and repeat the observation, when the following limb leaves it: now from these two tangential contacts the sun's azimuth may be calculated,

as shown before. The mean of the two will evidently be the true azimuth of the distant station, and thus we can lay down a true meridian as before.

If we also take the magnetic azimuth by a compass, the difference will be the variation.

When many detached plans of foreign countries are to be united into one general map at home, the directions of the magnetic and true meridians become of great importance; and we have seen, that unless equal altitudes are used for that purpose, the latitude is an essential element in their determination; for which reason we have, without entering into the niceties of geodesical measurement, or putting into requisition large and expensive instruments, shown how a good approximation may be had by the most simple method; and we have avoided all those which may be found in books written expressly upon astronomical, geodesical, or nautical subjects. In the following description of a well-known method of finding the longitude, our only object is to provide the reader with a specimen, in order to show that the instruments he is supposed to possess will also accomplish this useful purpose, by very simple means: thus proving that military men may have many excellent opportunities of increasing the general stock of knowledge as a matter of mere amusement, or while engaged in that branch of military duty of which we are now treating. In short, no foreign plan can be considered perfect, without the latitude and longitude of at least one or more principal places, within a

little of the truth, and the directions of the true and magnetic meridians nearly.

The easiest method of finding the longitude, next to that by time-keepers, is certainly by the eclipses of Jupiter's satellites; and it is more particularly applicable on shore. They happen so very frequently, that many observations may be collected in a short time, and will produce a good approximation.

We may remind our readers, who may not have studied this subject, that the difference of longitude between any two places is the angle at the pole contained between their two meridians, and is counted upon the equator: it is estimated either in time or space, and generally obtained in time, at least with respect to places very remote from each other, in which case it is the only method known: if, therefore, any instantaneous phenomenon is observed at two places, not under the same meridian, the difference between the time it happens at one place, and that at the other turned into space at the rate of 1 hour to 15° , will be the difference of longitude required; westward, if the time is earlier, and eastward, if the time is later: thus if it be 10 P. M. at one place, and 12 at the other, then the first mentioned is westward of the second, and the difference of longitude is 30° .

The eclipses of Jupiter's satellites are calculated in the Ephemerides for every month, and those visible at Greenwich are marked with an asterisk: but the unmarked ones will be visible in other parts of the globe, and are, therefore, useful to travellers, al-

though those visible at Greenwich are preferable; because, if observed there, the comparison of such observations made on other parts of the globe will give better results than a comparison with the mere calculated times, which are affected by the errors of the tables.

Now although not instantaneous phenomena, for they take several seconds to emerge or immerge out of or into Jupiter's shadow, yet the first, which is the quickest, is much used for this purpose, the instant it begins or ceases to be seen being accounted that of the phenomenon.

It is evident, that by simply observing an eclipse of one of these bodies, we identify a certain point of absolute time with the same instant observed, or which might be observed, at another place; and that by knowing the relative time at both places, the necessary element for finding their difference of longitude, or rather that difference itself, is obtained: for it simply requires to be turned into space, or the measure of degrees, &c., and the operation is complete.

A good telescope, not less than three feet in length, magnifying from forty to one hundred times, will enable us to observe the eclipse very well, if steadily supported, for it must not be held in the hand. A common watch, with a seconds' hand, must lie upon the table, and an assistant count the seconds carefully, unless a pendulum clock can be used; but we are now showing how to act with instruments not difficult to procure. The observer stops his assistant

at the instant the eclipse takes place, and quickly looks at the minute last past, and then at the hour, which he registers carefully. Immediately afterwards he must take the altitudes of one or two stars east and west of the meridian, the assistant counting time for him, and the instant of getting an altitude being noted exactly as was that of the eclipse. Now by applying the same well-known formula we used for ascertaining the azimuth, the true mean time of each of these observations can be found, and hence the error of the watch. This error being ascertained so soon after the observation of the eclipse, we may very safely consider it as having gone uniformly during the observations; and therefore its error applied to the time of the eclipse will give that time, by comparing which with that given in the nautical or other ephemeris, we at length get the difference of longitude.

The following specimen of an observation of this kind has been taken, as one done under circumstances of great privation of instruments, having only a small theodolite, reading to three minutes, and the sun's right ascension being calculated, instead of being taken at once from an ephemeris, which adds to the trouble :

1820. Nov. 1. — Emersion of Υ 's first sat. observed at 8^h 0^m 10^s by watch.

	h	m	s
Time by mean of four, observed } 8 20 34.5			
alt. of α ceti - - - }			

			°	'	''
Mean of four alt. of *	-	-	19	46	0
Refraction	-	-	-	2	37
True alt.	-	-	19	43	23
Complt. or Zen. dist.	-	-	70	16	37

For the star's culminating:

				h	m	s
Mean R. A. of * Nov. 1. 1820	-	-	-	2	52	54
— ☉ R. A. at time of observation	-	-	-	14	27	43
Apparent time of * culminating	-	-	-	12	25	11
— equat. of Time	-	-	-	-	16	16
True time of * culminating	-	-	-	12	8	55

For the time:

Z *	70	16	37			
Z P	38	39	54 ar. co. sin.	0.2042828	
P *	86	37	8 ar. co. sin.	0.0007566	
2)195	33	39				
	97	46	49			
— Z P	38	39	54			
	59	6	55 sin.	9.9335894	
$\frac{1}{2}$ sum } — P *	11	9	41 sin.	9.2868454	
					2)19.4254742	
$\frac{1}{2}$ P	31	4	16 sin.	9.7127371	

4 ^h	8 ^m	34 ^s	8'''	hour angle
12	8	55	0	* culminates
8	0	20	52	time of observing *'s alt.
8	20	34	30	ditto by watch
0	20	13	38	error of watch +

For the difference of longitude :

						h	m	s	
Em. Jupiter's first sat. observed	-	-	-	-	-	8	0	10	
— error of watch	-	-	-	-	-	0	20	13.68	
						<hr/>			
True time of emersion	-	-	-	-	-	7	39	56.32	
Ditto at Greenwich by Almanac	-	-	-	-	-	7	42	21	
						<hr/>			
Difference of longitude in time, west, because	it was earlier					}	0	2	
it was earlier									}
						<hr/>			
Or in space	-	-	-	-	-	0	36	10.20	
Correct longitude, by trigonometrical survey	of England					}	0	45	
of England									}
						<hr/>			
Difference too little by about six miles upon	the parallel of $51\frac{1}{2}^{\circ}$					}	0	9	
the parallel of $51\frac{1}{2}^{\circ}$									}

This may be considered as a very rough approximation, so many little corrections being left out, and the instruments employed in determining the time being so very imperfect.

The right ascension of the star was not corrected for aberration and nutation, as they are now ready for use in the Nautical Almanac; the whole being merely intended to exhibit the error taken at the worst; yet they rarely find their longitudes at sea to a greater degree of accuracy by the lunar method.

It appears unnecessary to mention other methods than that by a timekeeper, as we do not intend to enter deeply into a subject already so well handled by eminent and experienced authors; but only to provide an example or two for the use of those who may, from many circumstances, be very sparingly supplied with instruments and books.

If the observer has a timekeeper, he only needs

to determine the time at the place by the foregoing, or any other method, and to compare it with his time-keeper (which should give the Greenwich time) corrected by its daily rate; the difference is the longitude at once.

And when he has ascertained his longitude by the former operation, he will know how to apply a correction for the right ascension and declination of the sun, depending upon that longitude, with which, by going over his calculations again, he will get it more correct; for if he has 90° of longitude, the sun's right ascension and declination will evidently be different when it reaches his meridian, from what it was when it last passed that of Greenwich: and thus he provides himself with the element of that correction ready for future occasions.

The latitude of a place may be inferred from that of one exactly east or west of it, at a distance not exceeding five miles, and the longitude at any visible distance when upon the same meridian. This practice was followed by M. Gauthier, Captain of a French frigate, in the Mediterranean, and is a hint worth the notice of travellers, especially at sea, or wherever a geographical situation is inaccessible, as the top of a mountain, volcano, &c.

In the foregoing pages observations have been given and calculations made, for the mere purpose of showing what useful approximations may be derived from the use of small instruments when no better are at hand; but in all such cases it will be prudent to add a note on some part of the plan, stating what in-

struments were used, in order that it may be known what degree of confidence can be placed in them.

As general maps are commonly upon a scale of many miles to an inch, and as a second of a great circle of the earth is about 101 feet, it follows that an error of a mile in latitude or longitude would not be a serious objection, as far as they can be used in the construction of geographical maps, in the making of which we have said they are the principal materials.

CHAPTER VII.

LEVELLING BAROMETRICALLY, OR BY SPIRIT-LEVEL.

THE altitudes of objects may be ascertained by trigonometry*, by levelling, or by the barometer. The trigonometrical method will be familiar to most persons: that by levelling we shall now show, first describing the spirit-level; and shall give a short account of the barometrical method, accompanied by a real example of its application.

Levelling by the instrument we are now to describe is chiefly employed where great nicety is required, as in fortifications, forming canals, &c. This instrument (*Plate I. fig. 3.*) consists of a telescope of about two feet in length, mounted in supports, like those of the theodolite, and having a long spirit-level underneath it, and a compass to take bearings if necessary. It has also an adjusting-screw under the end next the eye, and the whole is mounted upon parallel plates; and three legs exactly like the theodolite, the rectifying of which alike applies to the spirit-level. It will readily occur to the reader, that it is more steady than the theodolite, it having no vertical limb, which might alter during an operation: notwithstanding this, the latter instrument may be

* Roughly, if by small instruments, without all the minute corrections.

used for levelling, if great care is taken; but then there should be a clamping-screw to the vertical arc.

There are a set of rods or staves (annexed figure) used with the level, capable of being joined together when great length is required; they are divided into feet and hundredths, or feet, inches, and tenths, and have a sliding vane, with a cross wire over a hole in the centre: this vane is moved up or down by the assistant, according to a signal preconcerted, until the cross wire agrees with the horizontal wire of the level, which has been previously set up and carefully rectified at the first station. The height of the centre of the levelling telescope being taken by one staff, and the height of the wire in the vane noted by the assistant, it is obvious that their difference, if any, is the difference of level between the two stations, the highest being always that which gives the smallest number of feet and inches.

But the distance between the stations is also to be known, and when this measurement is carried through many stations, we can draw a true vertical section of the ground, and this is the manner of making the correct sections required in constructing canals, and other works of that nature.

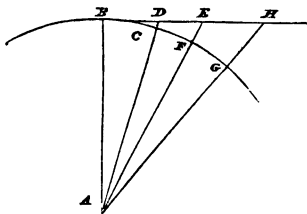
It is very clear that the more nicely we ascertain the distances between the stations, the better sections we shall produce; and these being always drawn on a large scale, it is of some consequence to avoid



measuring over very irregular ground, and the reductions incident to it. Now, when many stations occur, the practice is to set up the level at every other station, and level backward and forward to a couple of rods, thus saving much time.

The earth not being a plane, this method of levelling requires some correction, on account of the earth's sphericity; for the spheroidal form does not appear to enter into this subject materially in the short distances to which levelling stations usually extend, and the line of apparent level is, in fact, a tangent to the earth's surface at each station, the height of the instrument being nothing in comparison with its semi-diameter, and therefore it rises above the true level, which is at equal distances from the earth's centre every where, considered as a sphere, in a certain proportion to the distance: for this reason the correction must be applied to all distances exceeding 300 yards.

Let $BCFG$ (annexed diagram) be a portion of the earth's circumference, and BC , CF , FG , be considered as right lines, which they are as to sense, in any distance commonly required in levelling; then CD , EF , and HG , are the differences between the true and apparent level.



Now the difference between the true and apparent level, or CD , may be found at any distance BC , or,

which is very nearly the same, BD ; for by the well-known property of the circle $(2 AC + CD) : BD :: BD : CD$; but the diameter of the earth is so great, in proportion to the line CD , at the usual levelling distances, that $2 AC$ may be considered as $= 2 AC + CD$ in the above proportion without sensible error, and it becomes $2 AC : BD :: BD : CD$, or

$$\frac{BD^2}{2 AC} = \frac{BC^2}{2 AC} \text{ very nearly. Hence the difference}$$

between the true and apparent level is equal to the square of the distance between the stations divided by the diameter of the earth, and, therefore, is always proportional to the *square of the distance*.

But the diameter of the earth being nearly 7916 miles, if we make BC one mile, then the excess

$$\frac{BC^2}{2 AC} \text{ becomes } \frac{1}{7916} \text{ of a mile, or 8 inches, for the}$$

difference of the true and apparent level at 1 mile, the latter being the greatest; and, therefore, these corrections must be subtracted from the heights given by the instruments, when the distances between the stations render it necessary. The earth's mean diameter, in feet, is 41,796,480, or 7916 miles, as above, according to Laplace; and if we double the logarithm of the distance in feet, and add the ar. comp. of the logarithmic diameter, or 2.3788603, we shall get the difference of true and apparent level in feet, as in the following tables:—

	Feet.					Feet.	Inches.
B C	1000	-	-	-	-	0	0.29
$\frac{1}{4}$ mile	1320	-	-	-	-	0	0.50
	2000	-	-	-	-	0	1.15
$\frac{1}{2}$ mile	2640	-	-	-	-	0	2.00
	3000	-	-	-	-	0	2.58
	4000	-	-	-	-	0	4.59
	5000	-	-	-	-	0	7.18
1 mile	5280	-	-	-	-	0	8.00
	6000	-	-	-	-	0	10.34
	7000	-	-	-	-	1	2.07
	8000	-	-	-	-	1	6.37
	9000	-	-	-	-	1	11.25
	10000	-	-	-	-	2	4.71
	11000	-	-	-	-	2	10.74
	12000	-	-	-	-	3	5.34

	In Miles.					Diff. Level, Feet and Decimals.
B C	2	-	-	-	-	2.668
	3	-	-	-	-	6.003
	4	-	-	-	-	10.671
	5	-	-	-	-	16.674
	6	-	-	-	-	24.012
	7	-	-	-	-	32.683
	8	-	-	-	-	42.688
	9	-	-	-	-	54.027
	10	-	-	-	-	66.700
	11	-	-	-	-	80.707
	12	-	-	-	-	96.048

Many other uses may be found for this table, if sufficiently extended, in determining the extent of the visible horizon from a given height, and the height of distant hills, when exactly in the horizon; but there is yet another correction which is variable, and would affect the accuracy of such results as relate to remote objects, although not very sensible in

near ones, — it is the horizontal refraction, properly called terrestrial refraction, to distinguish it from the astronomical refraction, used in taking altitudes of the celestial objects.

In levelling operations it appears to require the height of the apparent above the true level to be diminished by about $\frac{1}{4}$ th of itself, when the distances are above half a mile. But this correction is variable, and the true refraction, at any given time, can only be ascertained by simultaneous observations with capital instruments, as may be seen by reference to the publications relating to the British survey and many other works.

Suppose it is required to draw a profile of the ground 1, 2, 3, 4. (*Plate II. fig. 4.*) The level is first set up and adjusted at A, a rod is placed at 2 and 3, and the assistants raise or depress the vanes, by signals given to them, until the centres correspond with the horizontal wires of the telescope; the distances 2 A and A 3 are then measured, and the whole entered in a book, or upon a rough sketch made on the spot. The instrument being removed to B, a similar proceeding takes place; and so on through the whole line. Another line and height may be taken at 1, if necessary, by removing the rod, and measuring the distances 1, 2. When these lines differ but little from the horizontal lines, they may be taken for them; but if the slope is very great it is easy to reduce them, because we have always the hypotenuse and perpendicular of a right-angled triangle given by our measurement to find

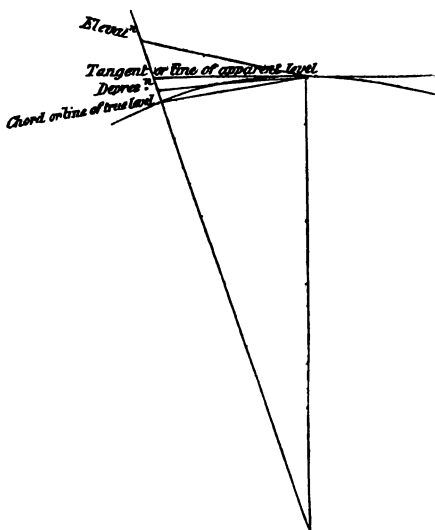
the base. But it is evidently impossible to level in this manner, without numerous stations on the side of very steep hills; for which reason the relative heights are better taken in the manner shown hereafter, by elevations and depressions.

By the preceding table it appears that if any of the distances between the stations amount to half a mile, two inches must be subtracted from the difference of level given by measurement, because the tangent or line of collimation rises so much above the true plane; and thus of all other long lines.

The design of this part of the work being to give instructions for ascertaining the difference of level near enough for most practical purposes, by employing portable instruments, we do not enter into the method of determining the refraction, but limit ourselves to such cases as most commonly occur, and where it is of little importance in military operations. We suppose the distances not to exceed two or three miles, upon which, as the refraction varies from about $\frac{1}{7}$ th to $\frac{1}{8}$ th of the contained arc, the elevations and depressions will only be affected by a few seconds, and the instruments we use are not expected to read nearer than $1'$, or perhaps $30''$: hence, as we merely seek an approximation, we may safely neglect it. Now, in such short distances, we may suppose them to be the bases of right-angled triangles, and the elevations or depressions to be the perpendiculars; therefore the preceding formula must be first applied for the difference between the true and apparent level. Then, because two places are upon



the same level when the depression is equal to half the intercepted arc of a great circle drawn from one to the other, or when the difference of true and apparent level is equal to the tangent of the depression, if the tangent of the depression *exceeds* the difference of true and apparent level, that difference must be subtracted from it, and the remainder will be the true difference of level, the distant place being lowest. And when the tangent of depression is *less* than the difference of true and apparent level, it must be subtracted from that difference, and the remainder will be the true difference of level, the distant place being highest.



When the distant place is on the line of apparent

level, it will be highest by the difference between the true and apparent level; and in all elevations the tangent is to be added to the difference of true and apparent level, and the distant station is highest. (See diagram opposite.) To this we may add, that in all depressions the height of the instrument is to be *subtracted* from the *true* difference of level, and when the distant place is in the line of apparent level; also in all elevations the height of the instrument is to be added to the *true* difference of level.

It generally happens that, from some elevated station, we know the distances to a great many objects on our sketch, and we have the logarithms of them ready for use. Such a station will be best for the purpose of finding the relative heights of many stations at once: for if we observe the elevations and depressions of those objects, we can calculate each separately, and thus from one or two heights we get a series of levels over a very extended space. It is proper to check some of them, at least from another spot, to prevent considerable errors from passing unnoticed: or we may find our station upon any of the under features, and take angles of depression or elevation to any object considered as a standard height, and which may be determined above the level of the sea or otherwise.

Specimen of calculation. (*Plate II. fig. 5.*)

A, B, and E, are three stations, the distances of which are known.

At B, E was in the apparent level,

And A was depressed, $0^{\circ} 9'$.

B E log. - -	4.1485958	B A log. - - -	3.1395614
	2		2
B E ² log. - - -	8.2971916	B A ² log. - - -	8.2791228
Const. log. - -	2.3788603	Const. log. - -	2.3788603
4.7429 E highest	0.6760519	4.5497 ap. dif. lev.	0.6579831
		B A log. - - -	4.1395614
		tan. 9' log. - -	7.4179696
		36.102 - - -	1.5575310
From B to E, the tan. is 0; the difference there- fore stands by itself - - - - -			4.7429
From B to A the tan. of depression is greater than the difference of apparent level, which must, therefore, be subtracted from it, or 36.102—4.5497 - - - - -			31.5523
Simple difference of level, one being an elevation and the other a depression - - -			36.2952

But to find the absolute heights with respect to the place of observation, we must add the height of the instrument, 4.1250 feet, to the first, and subtract it from the second, thus:—

$$\begin{array}{r}
 4.7429 \\
 + \quad 4.1250 \\
 \hline
 8.8679 \text{ E above B} \\
 \text{And } 31.5523 \\
 - \quad 4.1250 \\
 \hline
 27.4273 \text{ A below B}
 \end{array}$$

And $8.8679 + 27.4273 = 36.2952$ as before.

Another mode of correcting for the curvature of the earth is as follows:—

As equatorial radius in feet, 20919360 (ar. co. log. 2.6794516 constant)
: distance in feet
: : arc equal to radius in seconds, 206264".8 (log. 5.3144251 constant)
: arc of a great circle between the two places in seconds.

Hence, if we add the log. distance in feet to the above two constants, or their sum, and omit 10 in the index, we shall obtain the correction of $\frac{1}{2}$ the intercepted arc, or depression of the chord below the tangent.

When the depression is $\frac{1}{2}$ the intercepted arc, the two places are on the same level.

When an elevation, $\frac{1}{2}$ the intercepted arc must be added to it for calculation.

When a depression, the difference between $\frac{1}{2}$ the intercepted arc and the angle taken will be used for calculation.

When the depression exceeds $\frac{1}{2}$ the intercepted arc, the correction must be taken from the angle of depression, and the remainder used for calculation.

By first method.

Dist. 12493.7	log.	4.0966911	
			2
		8.1933822	
const. log.	+	2.3788603	
			ft.
app. dif. level	3.7346	0.5722425	
			tan. 13'.53"
+ log. distance		4.0966911	
			ft.
elevation	50.456	1.7029133	
+	3.7346		
			ft in.
	54.1906	or 54	2.2872
+ height of instr.		13	6.3
hill above Observ ^y .		67	8.5872

By second method.

Dist. 12493.7	log.	4.0966911	
const. log.	-	7.9938767	
		123."18	- 2.0905678
		or 2' 3."18	
		$\frac{1}{2}$ 1' 2" dep. chord	
elev.	13 53		
	14 55	tan. 7.6374006	
+ log. dist.		4.0966911	
			ft.
	54.212	1.7340917	
			ft. in.
		or 54	2.544
+ height			
of instr.		13	6.3
		67	8.844 nearly,
			as before.

As a more expeditious method of levelling has

been introduced since the vast works of late proceeding in this and other countries have demanded it, we shall give a slight sketch of it in this place, accompanied by the necessary illustrations.

The construction of the levels themselves has been improved: Mr. Gravatt's is considered the best. The telescope is but 10 inches long, and, having a wide aperture, admits more light, and consequently a greater magnifying power: it seems also likely to keep its adjustments longer, which is a great advantage, as these with most other surveying instruments ought always to be ready for use, and not require allowance for error, if possible to avoid it.

Levelling is in fact vertical surveying, and requires a base, with check-points, called bench marks, like marked stations in horizontal surveying, to start from or to close upon.

Whatever levels are used, they must be accompanied by staffs: these must be strengthened at the feet by brass or iron.* Those now generally used are in three lengths, altogether amounting to 12 or more feet, whether joined like a fishing-rod, or sliding out like a telescope; and since short distances must be employed, both to avoid refraction and curvature, they are divided into 100ths of a foot by alternate black and white lines, 10ths being generally added to assist the eye, while the figures, which are

* Formerly, pickets were driven even with the ground to rest the staves upon, but now pieces of metal are placed upon the earth and taken forward, or shoes of different contrivances are used by many persons.

not less than one inch in height, are marked very distinctly in red, being inverted to suit the telescope, which also inverts.

At 2 chains distance, the 100th of a foot can be bisected in steady weather.

5 chains they can be seen.

6 or 7 chains, and a little beyond, $\frac{1}{10}$ of a foot may be seen.

Now, according to the published tables —

	ft.		ft.		ft.	
At 5 chains	.00261	curvature,	.00037	refraction,	.00224	} both the latter.
10 "	.01042	"	.00149	"	.00893	do.
20 "	.04169	"	.00596	"	.03573	do.
And at 500 feet	-	-	.00513	} refraction and curvature.		
1000	-	-	.02059			
2000	-	-	.08203			

Thus we see that as at 5 chains the divisions can only be seen, and the whole correction amounts to no more than .00224 feet, it is not appreciable, and levelling stations may run as long as 10 chains very safely; indeed, in some varieties of country they will be very much shorter.

This being understood, the process is very simple: let the foot of the levelling-staff be placed on a bench-mark or other place, water of a lake, bank of a river, &c., and going forwards about 10 chains, or less if great accuracy is required, adjust the instrument, until, in turning it all round, the bubble remains in the centre: now enter the height where the cross wire cuts the staff in the 1st column, marked *back*; then sending the assistant forward as many paces or chains as he was behind, let him plant his staff again, and

TABLE I. (See Fig. 1.)

LEVELS FROM BLACKWATER BRIDGE TO THE COLLEGE LAKE.

Back.	Fore.	Rise.	Fall.	Reduced Levels from Datum.	Distances.	Locations.	Remarks.
6.50	3.18	3.32		3.32		Starling of Black-water Bridge. Road at Field Gate. Corner of Footpath in Oak Grove.	
5.90	4.71	1.19		4.51			
4.60	0.40	4.20		8.71			
6.27	0.32	5.95		14.66		Door-step of House.	
14.00	0.27	13.73		28.39			
13.92	11.23	2.69		31.08			
15.33	0.17	15.16		46.24		Floor of No. 5 Hall.	
10.43	2.50	7.93		54.17			
2.50	14.60		12.10	42.07			
5.30	14.20		8.90	33.17		Level of Water in Lake.	
3.07	8.50		5.43	27.74			
87.82	60.08	54.17	26.43				
60.08		26.43					
27.74		27.74					

TABLE II.—SPECIMEN BY THEODOLITE. (See Fig. 2.)

Measured Distances.	Elevation or Depression.	Calculation of Reduction to the Horizon.	Horizontal Distance in Links.	Calculation of Vertical Distances.	Relative Altitude in Feet.	Altitude above Low-water Mark.	Remarks.
944	1° 35' Ele.	2.974972 9.999834 2.974806	943.64	9.819544 2.974972 8.441394 1.235910	17.21		(1)
1649	1° 25' Ele.	3.217221 9.999867 3.217088	1648.5	9.819544 3.217221 8.393101 1.429866	26.90		(2)
446	2° 30' Ele.	2.649335 9.999586 2.648921	445.57	9.819544 2.649335 8.639680 1.108559	12.84		(3)
454	3° 0' Ele.	2.657056 9.999404 2.656460	453.37	9.819544 2.657056 8.718800 1.195400	15.68		(4)
636	2° 30' Ele.	2.803457 9.999686 2.803043	635.4	9.819544 2.803457 8.639680 1.262681	18.31		(5)
900	2° 45' Ele.	2.954243 9.999500 2.953743	898.96	9.819544 2.954243 8.681043 1.454830	28.50		(6)

The third column contains the log. co. sin. of the angle of elevation or depression, and the log. of the distance. The fifth column contains the log. of 66 = 9.819544, the proportion of one link to one foot, and the log. sin. of the angles, and log. of the distance; their sum gives the altitude in feet.

Fig. 1.

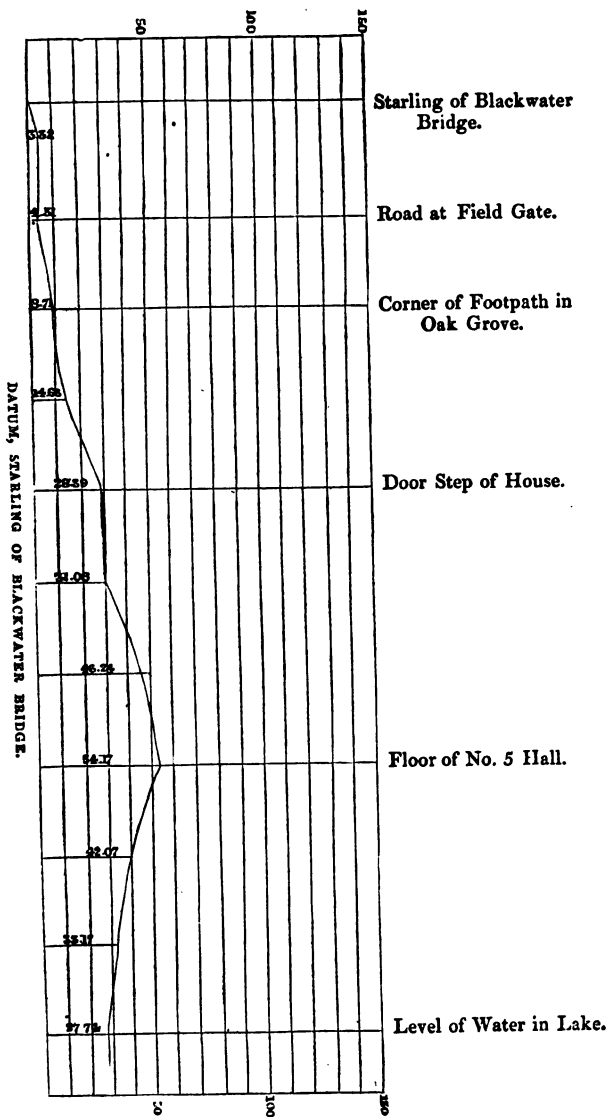
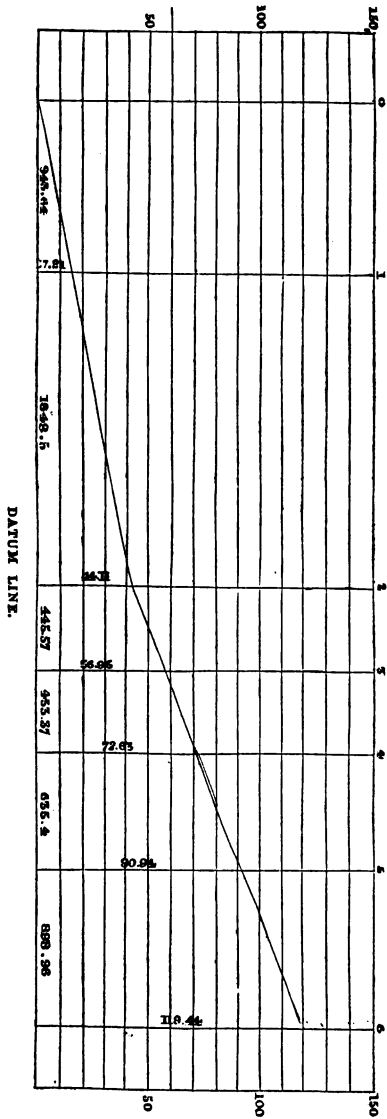


Fig. 2.



enter the height in the 2nd column, marked *fore*. Now the difference must be marked in the 3rd col. if a rise, which will be when the back station reads the greatest number; and in the 4th col. when the reverse, or the fore observation the greatest; by the continual addition of the rises we obtain the reduced level, but when descents occur they must be subtracted from the rises, and the reduced levels will be thereby obtained. If the second column be subtracted from the first, and the difference between the 3rd and 4th be taken, they will, if no error has been committed, be equal to the 5th, or reduced level. The distances between the stations should be measured, if the section is intended for any other purpose than merely ascertaining the difference of level between any two places, in which case any irregular line of route may be taken.

To lay down a section of this kind nothing more is necessary than to draw the datum line, and, raising perpendiculars at each measured distance, take their number from the column of reduced levels, and unite the lines for the true sections. Many persons prefer ruling parallels at every 10 feet, as in the specimen, numbering every 50, when only the remaining units are necessary to be taken from the scale. We have a section of more than 50 miles taken and laid down by the means pointed out for a railway, which was executed at a cost of about 1,000,000*l*.

In such expensive undertakings every precaution must be taken to ensure accuracy, but as several

persons are usually or may be employed on an extensive work, they should each begin at a common bench-mark, and work back again to the same: $3\frac{1}{2}$ miles have been carefully done from one, with a return to it, in less than a day, differing no more than $\frac{1}{100}$ th of a foot, and the sections laid down before night. Any permanent object serves for a bench-mark to be left off upon, and the staff set upon it, on resuming the work, may be continued in the regular manner at any time, by taking the instrument one station forward. If several bench-marks are left, and a section taken from one to another without measuring the distances, they will afford a good test of the intended original sections; and as this country is so much intersected by canals, the different levels of which are known, a series of tests can be obtained of great use, and has been done: Walker's Geological Map of England, &c. contains their levels. As this work is not intended for railway surveyors, we may be forgiven for giving general ideas, rather than enter into the extreme minutiae, as Mr. Bruff and others have done, and we may refer to Lieut. Frome's excellent work on Trigonometrical Surveying with confidence, as well as the also excellent text-books lately published for the use of the students here by Professors Narrien and Scott; for the work would be extended beyond all due bounds, as to size and price, were we to avail ourselves of the many and various books, English and foreign, that have been published on the subject of topographical surveying and drawing.

Where sections are required, as they often are for military purposes, amongst hills very abrupt, they may be taken by a theodolite divided to minutes or half minutes. If this is set up, and exactly levelled at some assumed datum level, and its height measured, that is to the centre of the telescope supports, and that height carried forward till another ascent or descent occurs, then an elevation or depression and the distance between being taken will afford data for a section, according to the field-book and diagram given, the first column of which contains the distance measured; the second, the elevation or depression; the third, the log. cos. of the angle and log. of the distance; the fourth, the horizontal distance in links; the fifth, the constant log. of 66 for converting the links into feet, the log. distance, and the log. sin. of the angle; the sixth, the relative altitude in feet; the seventh, the altitude above low water mark, or any other data; and the last column, any remarks that may be necessary. The whole can be laid down as before, the horizontal and vertical distances being given by the field-book. We have said nothing about reciprocal angles backwards and forwards at each station, because we have all along advocated the practice of keeping instruments in proper order for service whenever wanted, or adjusting them before use; time may be thus consumed, but it will be saved in the end: further information, to the minutest detail, will be found in Mr. P. Bruff's Engineering field-work, and in Lieut. Frome's, before mentioned.

We now proceed to the barometrical method of deducing the height of hills*: it is founded upon the principle that the density of the air decreases as we ascend upwards in a geometrical progression, the increment of ascent being in an arithmetical progression, as exhibited in the following table:—

Height in Miles.	Number of times rarer.
0	1
3½	2
7	4
14	16
21	64
28	256
35	1024 &c. &c.

The barometer being an instrument which shows the density of the air at any given time and place, it follows, that if two barometers are placed at different elevations above the level of the sea, then simultaneous observations upon them will give the density at the two places, and the law of progression being known, the difference of altitude is also known.

The formula, in the case we have supposed, is very simple; but as the air is rarefied by the heat, which is sometimes very different on different levels at the same time, and as the length of the barometric column of mercury is also affected by this circumstance at the rate of .00296 English inches for every degree of Fahrenheit's thermometer according to De Luc, it

* An instrument ready for use is to be had at Newman's, optician, Regent Street, which, by a simple calculation and by boiling water at the upper and lower stations, seems to replace the barometer admirably in measuring heights.

becomes necessary to have an attached thermometer to show the actual heat of the instrument, and a detached thermometer for that of the circumambient air in all cases; and by these circumstances the formulæ become a little more complicated.

The celebrated Biot has investigated the subject with much patience and labour: the result of his investigations has been a set of tables, in the construction of which every thing that could affect the formula has been ably considered, and the reader would do well to consult them, unless he prefers the following formulæ, either of which will give the height required: —

$$\{10000 \, l \overline{+}, 468 \, d\} \times \{1 + (f - 32^{\circ}).00245\}$$

by Gen. Roy,

$$\{10000 \, l \overline{+}, 440 \, d\} \times \{1 + (f - 32^{\circ}).00243\}$$

by Sir G. Shuckburgh,

where l = difference of logarithms of height of barometer,

d = difference of degrees of Fahrenheit's thermometer,

f = mean of the two temperatures, shown by the detached thermometer.

The sign — takes place when the attached thermometer is highest at the lowest station; and the sign + when it is lowest at that station.

The following is a real example by the first formula, where only one barometer was used; and although this is often done, yet, as the mercury frequently alters much in its altitude while going from one station to another, it is less to be depended upon than when two instruments are employed, which

have been proved to indicate alike, under like circumstances. The two formulæ have been found to agree within .7 of a foot, upon a height of 4037 feet, the latter giving the least number : —

Example.

	Barom.	At Ther.	Det. Ther.
Lowest station	30.24	56°.5	54°.75
Highest station	30.068	65 .1	65
		<hr/>	<hr/>
Difference		8.6 = <i>d</i>	2)119.75
			<hr/>
Mean -	-	-	59.875 = <i>f</i>

Hence $d = 8.6$

and $f = 59.875$

Log. 30.24 = 1.4805818

30.068 = 1.4781045

Diff. of logarithms 0.0024773 = l

Then by first formula $f - 32^\circ = 27.875$, and $1 + (27.875 \times .00245) = 1.07074375$; and $10000 \ l = 10000 \times .0024773 = 24.773$; also $.468 \ d = .468 \times 8.6 = 4.0248$.

Now 24.773

+ 4.0248

28.7978, which, multiplied by 1.07074, produces 30.83 fathoms, or 184.98 feet, the difference of level required.

The barometrical observations furnish the means of ascertaining the difference of level between places invisible from each other; and by comparing the mean results of several years with the mean level at the surface of the sea, or 30.035 inches, latitude 50° , at the mean temperature of the exterior air, and of the instrument 55.04 Fahrenheit, or whatever any other latitude may happen to give, we get the

difference of level over any extent of country we please.*

A standard elevation may at least be found by this means, and others deduced from it by those already detailed.

If to the foregoing particulars we add a few landscape sketches, or views of the more remarkable places, or general appearance of the country from some elevated spot, then the most complete idea of that country will be given to every person who shall see a plan so finished.

* The following results have been found from Sir H. Englefield's method and table, and compared with the same heights by levelling :—

		Feet.	Feet.
By barometer	- - -	62.84	and 150.13
By levelling	- - -	62.50	150.3
Feet.			
116.58	by Lieut. Frome from Mr. Bailey's formula.		
116.3	by Mr. Jones's tables.		
116.61	by Dr. Hutton's rule.		
116.93	by Sir H. Englefield's, rather roughly done by me.		
Snowdon by a mean, Col. Mudge	-	3561	} trigono- metrically.
Another mean	- - -	3568	
Barometer and Sir H. Englefield's tables,		3565.	

CHAPTER VIII.

FURTHER USE OF SEXTANT.

IT has been said in the preceding pages that reflecting instruments require an artificial horizon to take altitudes, but we cannot use them for small altitudes, because the rim of the vessel containing the reflecting fluid renders it impossible, nor can we take depressions by such means. Yet as a military man may have occasion for such observations, and not possess an instrument provided with a vertical circle and level, we shall show how this may be done nearly, that is to say, within two or three minutes, by a reflecting instrument. Place three strong stakes across, like the triangle used for hanging a kettle over the ground, binding them firmly at their junction; 'across two of the legs tie a fine thread tightly, and place underneath any vessel containing a fluid, as mercury or water. Now it is plain that when we look from above so as to bring the thread and its reflected image into exact coincidence, our eye will be in a vertical plane; therefore, by resting upon the stakes, and bringing the reflected image of a distant object into exact contact with the thread, we shall measure the supplement of the zenith distance, and if that is less than 90° , its

complement will be the depression; but if above 90° , the surplus will be the elevation. This apparatus can be made anywhere, and we insert this expedient in order to show that, with apparently slender means, we may always do something.

For the same reason we also insert a method of finding the heights of objects with a very little trouble by the sextant and the subjoined table: —

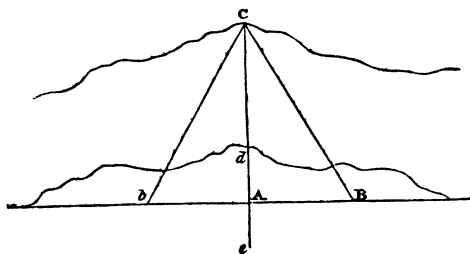
Multiplier		Angle.	Angle.	Divisor.
1	- -	45.0	45.0	1
2	- -	63.26	26.34	2
3	- -	71.34	18.26	3
4	- -	75.58	14.2	4
5	- -	78.41	11.19	5
6	- -	80.32	9.28	6
8	- -	82.52	7.8	8
10	- -	84.17	5.43	10

Rule. — Make a mark opposite the eye upon a wall or otherwise, and having set the instrument to any of the angles of the table most likely to answer, retire upon the ground, *which should be level*, until the reflected image coincides with the mark left upon the wall: now measure the distance from thence to the wall, and multiply or divide it, according as the table specifies; to this product or quotient add the height of the eye, and it will give the height required. The parallax of the instrument will not affect the result materially; it depends upon the hypotenusal distance, and may be found by adjusting the sextant to the top of the wall, &c., and observing how much it places the zero back upon the arc of excess; then the angle of the table should be reduced by that

quantity. A walking-stick, or even the length of the foot, will serve as a measure.

An example in illustration will be found in *Plate III. fig. 4.*

This method of taking heights by the small table may be used also for distances, where the ground is level enough to admit pacing or measuring a base; for if we set the instrument to any of the angles of the table, and pace or measure at right angles to a line joining our first station and the distant object, until it is seen in contact with a mark left at the point of departure, the distance required will evidently be a product or quotient of the base, according as we choose a multiplier or divisor: this may serve to determine the position of a battery, the breadth of a river, &c., with expedition, and without tables of logarithms.



It often happens that the most simple things are the most useful, and we shall show that in this case it is so. Suppose that on ground nearly level we require the distance of an inaccessible object *C* from *A*. If upon any support at *A*, a picquet or post, &c. we

place the sextant, having first set it to 90° , and direct the line of sight to *c*, causing an assistant by waving the hand right or left to place a camp colour, or a mere straight stick, so as to cover the object at *c*, as seen in the sextant, and then dropping one hand, he places the flag or other object firm and perpendicular in the ground. Then having judged what fraction of the distance *AC* can be paced or measured, *AB* on the angle is taken from the table, suppose $\frac{1}{3}$, and angle $71^\circ 34'$: setting the sextant to this angle, we pass on beyond the flag, and walking forward or backward, always keeping the flag and original support in line, we perceive that the distant object is reflected upon *A* at *B*; from this place we measure or pace the distance *BA*, and multiplying it by three obtain *AC*. Supposing that it was required to place a battery at 600 yards distance from *c*, fronting the line *AC*, and the distance *AC* was found to be 650 yards; then 50 yards measured from *A* to *d* would give the required distance; but if 700 yards, 50 yards from *A* to *e* would give the required place. The breadth of a river is found exactly in the same manner, but as they generally wind very much, it is better, if possible, to retreat inland, and subtract *Ad* from *AC* for the true breadth required. When a base can only be obtained in the opposite direction, *Ab*, the operation requires that we look towards *b*, and having brought the object *c* into the centre of the field by moving round, place the assistant so as to cover it, and on returning to *A*, keep the eye upon *c*, and go on till *A* is in line with *c* by reflection: the rest remains as before.

When an angle greater than 120° is required to be laid down, as the sextant seldom reads so far, and the uncertainty is then considerable, it is better to lay down the supplement in an opposite direction. The whole operation takes little more time to a practised person than to describe it.

This table may also be used to determine points upon table land rapidly, as right angles are easily constructed; and when one triangle has been formed, others may be also formed upon each side of it, if the ground admits.

The construction of instruments varies so considerably, that we can only attempt to give general notions upon their nice adjustment before they can be used in the field; and we shall begin with the theodolite. The levels are placed parallel to the horizontal plates and to the line of collimation by capstan-headed screws and a small lever; those on the horizontal plate should be first corrected, one at a time, by bringing it over, or rather parallel to, the levelling screws underneath, and adjusting it half by the one and half by the other. When the first level will go all round correctly, the other at right angles to it can be corrected by its own screws alone.

The vertical arc being set to zero admits the adjustment of the telescope-level more easily, because the horizontal plate is now supposed free from error.

The line of collimation requires that for its adjustment we should release the supports of the telescope, and allow it to turn freely around its axis; then as

the diaphragm carrying the cross wires is provided with four screws at right angles to each other, by releasing one and screwing the opposite one, we can bring the two wires into the axis of the telescope by trials, when it will be found that if we turn the telescope quite round upon its axis, the intersection of the wires will not describe a circle about a distant object, but remain fixed upon it in all positions of those wires with respect to the horizon.

Whenever we have reason to suspect the accuracy of the line of collimation, we should take altitudes with the level uppermost, and also underneath. If the horizontal wire is wrong, one will be as much too great as the other is too little, and the mean will be right: but if the level is false, we must, after one observation, take the telescope out of its supports, and, turning the horizontal limb half way round, replace the telescope and observe again; the mean will be right. These are the niceties we before spoke of, and the methods of avoiding errors in azimuths, &c., when we have not the means of carefully adjusting an instrument; but these adjustments are altogether distinct from the common ones by the finger-screws, in the ordinary use of this instrument. The spirit-level is corrected in a similar way.

It is essential that the mirrors of a reflecting instrument should be parallel to each other when it is set to zero, and also perpendicular to the plane of the instrument; they are therefore provided with proper screws to effect this adjustment.

In the pocket sextant, and all others, the great or

index mirror is correct by the construction ; the small one alone can require alteration. A small lever is connected with it, and moves it upon its centre of motion, by means of a screw in the circumference, which is turned by a key : this brings the two parts of a vertical object in contact, when the instrument is at zero and held horizontally ; but it should be at least a mile or two distant. The next correction will be made by applying the same key to an arbour upon the face of the instrument, while holding it horizontal, and looking at the distant horizon, both in the silvered part of the small speculum and also beyond it on the side ; for when the real and reflected horizons coincide, the mirror is perpendicular to the face of the sextant, otherwise not : this correction is best performed without the telescope, at first, because we then have a great field of view, and may be rendered more perfect by employing the telescope afterwards, which, by its magnifying power, will enable us to produce a better contact, not altering that at first obtained without it, unless necessary.

When these adjustments are perfect, if we put the dark glass upon the eye end, and the telescope is pointed at the sun, or without that dark glass if at the full moon, and by the finger-screw bring the direct and reflected images into perfect contact, the vernier will be at zero ; but when at zero, if the images pass each other either way, horizontally or vertically, the proper correction must be applied.

It is usual in large sextants and other instruments to find their error whenever they are used ; but we

prefer having instruments perfectly adjusted and ready for use: and we know, from experience, that with care they will long remain so, unless when they are so large as to alter from their weight, or are exposed to injuries by carriage or otherwise.

We doubt not that experience and study of the construction of instruments will be of infinitely more service to our readers than anything we can say upon the subject; nor need we speak of the adjustments of the reflecting semicircle, for its principles are so nearly the same as a sextant, that they are almost alike in both instruments.

Simms on Instruments, and many other books, will provide ample information upon this head, when the most perfect instruments are employed.

APPENDIX.

THE new French measures, and those of other countries, frequently occurring in maps and plans, the following tables, &c. are added, to save references to other books : —

French metre - - - - 39.371 English inches.

Ditto - - - - - 3.2809 English feet.

Ditto - - - - - 1.09363 English yards.

1609.32 metres - - - 1 English mile.

Old French foot - - - 12.78933 English inches.

„ „ toise - - - 6.394593 English feet.

100 degrees in a quadrant, according to the new division of the circle.

100 minutes to one degree.

100 seconds to one minute.

10,000,000 metres = 1 quadrant of the meridian.

100,000 „ = 1 degree.

1,000 „ = 1 minute.

10 „ = 1 second.

1 „ = .1 second.

To convert French metres into English feet. Multiply the metres and decimals by 3.28.

French metres into English yards. Multiply them by 1.09.

To convert the new degrees, &c. into the old division. Subtract one-tenth, thus :

$$\begin{array}{r} 73.1648380 \\ - \frac{1}{10} 7.3164838 \\ \hline \end{array}$$

65.8483542, which reduced will be
65° 50' 54''.07512, according to the sexagesimal measure.

To convert the old division into the new. Reduce the minutes, seconds, and decimals, to decimals, and add one-ninth, thus : —

$$\begin{array}{r}
 65^{\circ} 50' 54''.07512 \\
 65 \quad 50.901252 \quad \text{dividing the seconds by 60.} \\
 \hline
 65.8483542 \quad \text{dividing the minutes by 60.} \\
 + \frac{1}{9} \quad 7.3164838 \\
 \hline
 73.1648380 \quad \text{in the new division.}
 \end{array}$$

The French new method of adapting a scale to their maps or plans is by making it some definite fraction of the whole in linear measure, as $\frac{1}{80000}$ or $\frac{1}{100000}$, &c. : this is reduced to the usual English measure by merely dividing the number of inches in one mile, or 63360, by the denominator of the fraction thus :—Suppose $\frac{1}{80000}$, then $\frac{63360}{80000} = 10.56$ English inches to one mile; $\frac{1}{200000}$ will in like manner be $\frac{63360}{200000}$ or 3.168 inches to one mile.

The length of a degree of longitude upon any parallel of latitude is given by the following formula : —

Let L = given latitude.

Then as equatorial diameter is to the polar diameter, so is tangent of $90^{\circ} - L$ to tangent A ; and as rad. is to sine A , so is the length of a degree of the equator to the length of a degree on the parallel of the given latitude.

MEASURES OF VARIOUS COUNTRIES.

English statute miles in one degree	-	-	69 $\frac{1}{4}$
Scotch miles	-	-	61 $\frac{1}{3}$
Irish -	-	-	54 $\frac{1}{3}$
Geographical	-	-	60
Marine leagues	-	-	20
German miles	-	-	15
Dutch miles, or leagues	-	-	19

French leagues	-	-	-	-	-	25
Russian wersts	-	-	-	-	-	104 $\frac{1}{4}$
Spanish leagues, of 7572 varras	-	-	-	-	-	17 $\frac{1}{2}$
Portuguese leagues	-	-	-	-	-	19
Norwegian and Swedish miles	-	-	-	-	-	10 $\frac{1}{2}$
Danish miles	-	-	-	-	-	14 $\frac{3}{4}$
Prussian	-	-	-	-	-	16
Hungarian miles	-	-	-	-	-	13 $\frac{1}{3}$
Italian miles	-	-	-	-	-	60
Tuscan and Milanese miles	-	-	-	-	-	67
Piedmontese miles	-	-	-	-	-	48
Turkish agach, or league	-	-	-	-	-	22
Turkish berri	-	-	-	-	-	66 $\frac{1}{2}$
Grecian miles, used in the Archipelago, about						95 $\frac{1}{2}$
Grecian miles used in Turkey (rated at seven stadia,)						
each 4232 $\frac{1}{2}$ English feet	-	-	-	-	-	88
Mil Kebir, or great Arabic mile	-	-	-	-	-	50
Mil Sogair, an Arabic measure	-	-	-	-	-	95 $\frac{1}{2}$
Giam, an Arabic nautic measure, each = to nine						
great miles	-	-	-	-	-	5 $\frac{5}{8}$
Great parasangs, or Persian leagues	-	-	-	-	-	17
Common parasangs	-	-	-	-	-	25
Hindoostanee coss, Northern Provinces	-	-	-	-	-	36 $\frac{1}{2}$
Indian coss, in Delhi	-	-	-	-	-	34 $\frac{1}{2}$
Sultany coss	-	-	-	-	-	17 $\frac{1}{2}$
Mogully coss	-	-	-	-	-	41
Royal coss	-	-	-	-	-	26 $\frac{1}{4}$
Carnatic coss	-	-	-	-	-	37 $\frac{1}{2}$
Brammy, or Punjaby coss	-	-	-	-	-	56
Malabar Gau	-	-	-	-	-	10
Siam Roening varies from 20 to	-	-	-	-	-	33
Chinese modern li	-	-	-	-	-	334
Japanese leagues	-	-	-	-	-	33 $\frac{1}{2}$
Ancient Roman miles	-	-	-	-	-	75

As the arcs, sines, and tangents differ but little from each other to 4 places of figures, from 0' up to 8', the one may be substituted for the other in calculations where small arcs between these limits are used: this is, in fact, the doctrine of parallaxes; when a given object is at twice the distance, it subtends half the angle; and this reasoning applies in surveying on many occasions. An object subtending an angle of 1' can be seen by a good eye at 3437.74 times its diameter, or in round numbers 3450; hence many useful deductions may be made.

An error in taking angles for 1', right or left of an object, will throw it out 1.53120 feet at 1 mile.

3.06240	—	2
4.59360	—	3
6.12480	—	4
7.65600	—	5
9.18720	—	6
10.71840	—	7
12.24960	—	8
13.78080	—	9
15.31200	—	10

We now see that for distant intersections very exact angles must be taken, and that upon small scales imperfect bearings will do, for at 4 inches to 1 mile, 10' error would only amount to about $\frac{1}{132}$ of an inch in a mile; inferior instruments, therefore, may do for some purposes, but not without checks for much better ones.

To aid persons in choosing telescopes for particular purposes, who may be at a distance from London, the following table of Mr. Dollond's has been added (except No. 2., made by Mr. Cary, 181. Strand). Makers are in the habit of advertising them for 10, 20, &c. miles;

their powers in real service, from long personal observation, have been ascertained, and may be relied upon.

No.	Diam. in.	Obj. Glass.	Power.	Open. in.	Shut. in.	Price. £ s
1.	1.1		22	14	5.	2 2
2.	1.3		26	20	7.5	2 10(Mr.Cary.)
3.	1.6		35	28	9.0	3 3
4.	2.05		45	40	10	7 7
5.	2.75		55	52	14	13 13

No. 1. A useful pocket instrument, has shown a windmill, &c. beyond 12 miles.

No. 2. will show the difference between the structure of two or more windmills at 20 or more miles; also churches, ships, &c. at less distances, and takes flag signals at 10 miles, when not very complex.

No. 3. performs the foregoing service more perfectly.

No. 4. The same as the celebrated instruments furnished by Mr. Dollond to the Admiralty, when the first telegraphs were established in England, and which were placed at distances, sometimes exceeding 10 miles.

By these telescopes the minutest parts of the telegraphs could be perfectly distinguished at the usual distances, and they could have been worked much further off in clear weather. With telescopes of this size the planet Jupiter and his system can be well seen, with an extra power Saturn's ring, and probably two of his satellites. Beyond these, as at No. 5., the telescopes begin to be very useful in astronomy, but are too cumbrous for common use, though some persons will have them, and they of course are preferable to all the others. A good eye and favourable weather are supposed, and it will be found that in good instruments the power is not overrated; they all

perform better with only one drawer, are rather less expensive, but not so portable. The foregoing table may also be thus used : — a distant object, about 3 feet in diameter, will subtend an angle of $2'$ at 1 mile, but removed to 20 miles it would be but the twentieth part of $2'$, or $6''$; and if 9 inches diameter, it will subtend an angle of $30'$ at 1 mile distant, which at 10 miles becomes $3''$, and at 20 miles $1''.5$. If a signal 9 feet in diameter is 10 miles distant, the subtense at 1 mile being $6'$, the angle is diminished to $36''$; but a telescope magnifying the linear dimensions 30 times would produce nearly $18'$; No. 3. telescope would, therefore, answer the purpose very well. Bright objects are seen much further than any others, and General Roy says, that with reflectors of 10 inches in diameter and large burners, his lamps could be seen at 20 or 24 miles; the telescopes were 3 feet focal length, with 2.5 inch double object-glasses, and that those eyeglasses of the least magnifying powers were found both in day and night observations to answer best; the least were under 60. Drummond's light, Bude's light, and other means are now employed; they are visible at very great distances.

As the describing circular arcs of great and inaccessible radii may be serviceable, the following method occurred to the author. Straight lines are usually chosen by surveyors for their narrow strips, and this has been before described; but where a curve happens, the angle between one straight line and the next being obtained, and the distance from this angular point on either side where the curve is to commence, it must terminate at the same distance on the other.

Now in the diagram, if BEFC represents the curve, and BA and AC the portions of the straight lines meeting in A, then, supposing perpendiculars to be let fall from

scribed on the ground by going round it, and the following method appears easy.

EXAMPLE.

Let BA be	3960 feet.	then in $\triangle ABD$	
and angle BAC	$= 150^\circ$	$\tan. A.$	10.5719475
the angle at the } centre will be }	$= 30^\circ$	$+ \log. BA.$	3.5976952
and BAD	$= 75^\circ$	$-10 BD, 14778.9$	4.1696427
		or the rad. of the arc $BC.$	

Now supposing, to make the thing easy, we take the arc $BE = \frac{1}{3}$ of the whole, or 10° ; then the

nat. sin. of $\frac{1}{2}$ the arc, or $5^\circ = .087156$

	$\frac{2}{}$	
chord	.174312	$\log. \overline{1.2413273}$
	BD	$\log. 4.1696427$

The chord $BE \quad 2576.14 = 3.4109700$

This chord is to be laid off in every instance from B to E , E to F , &c., on the angles ABE , GEF , &c.

Thus. At B lay down half the angle BDE , or 5° , because the chord BE falls so much below the tangent BA , and measure out BE (in this case) 2576, in feet, which will give the first point in the curve E .

At E , upon BE produced towards G , lay down the angle $GEF = 10^\circ$, because EG rises as much above the tangent EK as the chord falls below it, and measure the same distance as before from E to F .

The same process repeated at F will give the remaining distance FC , and prove the work.

But these arcs and distances must be very small, and the same process still holds good.

Thus, supposing a radius of 1320 feet, or $\frac{1}{4}$ of a mile, and the arc 50° .

Now, if $\frac{1}{10}$ part of it be laid down at each station according to the fore-mentioned method, then the first

chord will fall below the tangent 30', and all the rest will make angles of 1° with the last chord produced.

And, Nat. sin. 30' = .008727
2

Chord	=	.017454, log. 2.2418950
Log. rad. in feet (1320),		<u>3.1205739</u>
Chord in feet	23,039	<u>1.3624689</u>

to be measured from each station to the one succeeding it: small arcs may have their logs. added to the rad. at once; the above circular arc is .017453, which would alter the last decimal figure, and all below it, still more nearly agree. But in setting off such curves, when they pass over steep ground, it may be necessary to reduce the measured lines where they differ much from their horizontal bases. If the curve takes a contrary flexure at its termination, the two will have a common tangent, although the radii may differ, and the radius of the second curve, either longer or shorter, admits of the same kind of calculation as the first.

The following railway slopes are shown by their angles to the nearest second: —

Greatest allowed, 1 in 36	1°	35'	28''
— 40	1	25	55
— 50	1	8	45
— 60	0	57	17
— 70	0	49	6
— 80	0	42	58
— 90	0	38	11
— 100	0	34	23
— 200	0	17	11
— 300	0	11	28
— 400	0	8	35
— 500	0	6	52

This table confirms what has been said respecting doubling, &c. distances when the angles are small.

Military slopes, 60° or 4 to 7, inaccessible for infantry.

45° or 1 to 1, difficult.

30° about 7 to 4, inaccessible for cavalry.

15° — 4 to 1, do. for wheel carriages.

or $4^\circ 45' 49''$ } — 12 to 1, easy for carriages.
nearly }

316.227 links = side of a square containing an acre, a very near approximation.

208.71032 feet = ditto, still nearer.

69.57 yards = ditto.

6.2773661 constant logarithm to convert feet into miles. The distances in feet are given by the triangles; therefore their log. added to the const., omitting 10 in the index, will stand thus:—

Const. = 6.2773661

Log. dist. 39443.2 ft. = 4.5959719

7.4703 miles. 0.8733380

Notwithstanding there has been no pretension to geodesical nicety in this work, some leading problems have been added by Professor Hearn, that the reader may be aware of the kind of triangulation required on very extensive surveys of kingdoms, provinces, &c.; and as the calculations of excavations are frequently in demand, a problem upon that subject follows the others: although in practice tables are constantly used for shortening the work, it is highly desirable that practical persons should be well acquainted with the principles on which they are founded.

IN SPHERICAL TRIGONOMETRY.

OBLIQUE-ANGLED TRIANGLES.

A, B, C, represent the angles, a, b, c , the sides respectively opposite them.

$$(1) \quad \frac{\sin. a}{\sin. b} = \frac{\sin. A}{\sin. B}$$

$$\begin{aligned} \text{or } \log. \sin. a &= \log. \sin. A + \log. \sin. b + \log. \operatorname{cosec}. B \\ \log. \sin. A &= \log. \sin. B + \log. \sin. a + \log. \operatorname{cosec}. b \end{aligned}$$

These formulæ are for the cases in which two angles and a side opposite one of them, or two sides and an angle opposite one of them, are given.

$$(2) \quad \tan. \frac{1}{2} (A - B) = \cot. \frac{C}{2} \cdot \frac{\sin. \frac{1}{2} (a - b)}{\sin. \frac{1}{2} (a + b)}$$

$$\tan. \frac{1}{2} (A + B) = \cot. \frac{C}{2} \cdot \frac{\cos. \frac{1}{2} (a - b)}{\cos. \frac{1}{2} (a + b)}$$

$$\tan. \frac{1}{2} (a - b) = \tan. \frac{C}{2} \cdot \frac{\sin. \frac{1}{2} (A - B)}{\sin. \frac{1}{2} (A + B)}$$

$$\tan. \frac{1}{2} (a + b) = \tan. \frac{C}{2} \cdot \frac{\cos. \frac{1}{2} (A - B)}{\cos. \frac{1}{2} (A + B)}$$

$$\text{or, } \log. \tan. \frac{1}{2} (A - B) = \log. \cot. \frac{C}{2} + \log. \sin. \frac{1}{2} (a - b) + \log. \operatorname{cosec}. \frac{1}{2} (a + b)$$

$$\log. \tan. \frac{1}{2} (A + B) = \log. \cot. \frac{C}{2} + \log. \cos. \frac{1}{2} (a - b) + \log. \sec. \frac{1}{2} (a + b)$$

$$\log. \tan. \frac{1}{2} (a - b) = \log. \tan. \frac{C}{2} + \log. \sin. \frac{1}{2} (A - B) + \log. \cos. \frac{1}{2} (A + B)$$

$$\log. \tan. \frac{1}{2} (a + b) = \log. \tan. \frac{C}{2} + \log. \cos. \frac{1}{2} (A - B) + \log. \sec. \frac{1}{2} (A + B)$$

These are Napier's analogies, and are useful when two sides and contained angle, or two angles and side between them, are given.

(3) Let $s = \frac{1}{2}(a + b + c)$

$$\sin.^2 \frac{A}{2} = \frac{\sin. (s - b) \sin. (s - c)}{\sin. b \sin. c}; \quad \cos.^2 \frac{A}{2} = \frac{\sin. s \sin. (s - a)}{\sin. b \sin. c}$$

or, $2 \log. \sin. \frac{A}{2} = \log. \sin. (s - b) + \log. \sin. (s - c) + \log. \operatorname{cosec}. b + \log. \operatorname{cosec}. c$

$$2 \log. \cos. \frac{A}{2} = \log. \sin. s + \log. \sin. (s - a) + \log. \operatorname{cosec}. b + \log. \operatorname{cosec}. c$$

These are the most generally useful formulæ when three sides are given.

(4) Let $E = A + B + C - 180^\circ$; then E is called the "spherical excess."

$$\cos.^2 \frac{a}{2} = \frac{\sin. (B - \frac{1}{2} E) \sin. (C - \frac{1}{2} E)}{\sin. B \sin. C}$$

$$\sin.^2 \frac{a}{2} = \frac{\sin. \frac{1}{2} E \sin. (A - \frac{1}{2} E)}{\sin. B \sin. C}$$

or, $2 \log. \cos. \frac{a}{2} = \log. \sin. (B - \frac{1}{2} E) + \log. \sin. (C - \frac{1}{2} E) + \log. \operatorname{cosec}. B + \log. \operatorname{cosec}. C$

and $2 \log. \sin. \frac{a}{2} = \log. \sin. \frac{1}{2} E + \log. \sin. (A - \frac{1}{2} E) + \log. \operatorname{cosec}. B + \log. \operatorname{cosec}. C$

These formulæ are useful when the three angles are given.

It is shown in works on spherical trigonometry that the spherical excess is the measure of the area of the spherical triangle to which it belongs.

The excess may be expressed in terms of the sides of the triangle by several formulæ.

LLHUIILLIER'S FORMULA.

$$\tan. \frac{E}{4} = \sqrt{\tan. \frac{1}{2} s \cdot \tan. \frac{1}{2} (s - a) \tan. \frac{1}{2} (s - b) \tan. \frac{1}{2} (s - c)}$$

CAGNOLI'S FORMULA.

$$\sin. \frac{E}{2} = \sqrt{\frac{\sin. s \cdot \sin. (s - a) \sin. (s - b) \sin. (s - c)}{2 \cos. \frac{a}{2} \cos. \frac{b}{2} \cos. \frac{c}{2}}}$$

Also if Δ be the area of the spherical triangle, and r the radius of the sphere, the number of seconds in the spherical excess is

$$E'' = \frac{\Delta}{r^2 \sin. 1''}.$$

From this formula I have found that the number of seconds in the spherical excess of a geodesic triangle whose area is 100,000 acres amounts to $2''.06$ very nearly. Hence, if in any geodesic operation the area of a triangle be approximately found (by considering it as a plane triangle), and that area be expressed in acres, the number of seconds in the spherical excess will be

$$.0000206 \times \text{area of triangle.}$$

Thus, if the triangle is found to contain 568240 acres

$$\begin{array}{r} 568240 \\ \quad 206 \\ \hline 3409440 \\ 1136480 \\ \hline 11.7057440, \text{ or } 11''.7 \text{ nearly.} \end{array}$$

As a very convenient rule, where *extreme* accuracy is not required, cut off the last four figures, expressing the area in acres, and divide by 5. When there are not more than four figures in the number, the excess is insignificant.

By means of the spherical excess we can estimate the sum of the errors of observation of the three angles of a geodesic triangle. For example, if the three observed angles are

$$\begin{array}{r} 65^\circ \quad 45' \quad 27'' \\ 72^\circ \quad 19' \quad 33'' \\ 41^\circ \quad 55' \quad 4'' \\ \hline 180^\circ \quad 0' \quad 4'' \text{ apparent sphl. excess.} \end{array}$$

Now suppose that from knowing also one of the sides we obtain (approximately) the area of the triangle, 256,750 acres, cutting off 6750, and dividing 25 by 5, we have $5''$ for the true spherical excess, and hence the difference in the true and ap-

parent excess is $1''$, which is therefore the sum of the errors of observation.

NAPIER'S RULES FOR RIGHT-ANGLED SPHERICAL TRIANGLES.

Although the formulæ for oblique-angled triangles are of course applicable to right-angled triangles, yet, when applied to the latter, they become greatly simplified, and may be exhibited in two concise rules easily remembered.

In right-angled spherical triangles the following are called the circular parts, the angle c being the right angle,

$$a, b, 90 - A, 90 - B, 90 - c.$$

These being arranged in a circular order thus,

$$\begin{array}{ccc} & a & \\ \text{co. } B & & b \\ & \text{co. } c & \text{co. } A. \end{array}$$

where $\text{co. } A$ signifies "complement of A ," we have

$\sin.$ middle part = product tangents of adjacent parts,

and $\sin.$ middle part = product cosines of opposite parts.

Thus,

$$\sin. \text{co. } B = \tan. a \tan. \text{co. } c = \cos. b. \cos. \text{co. } A$$

$$\text{or, } \cos. B = \tan. a \cot. c = \cos. b \sin. A$$

$$\sin. a = \tan. \text{co. } B \tan. b = \cos. \text{co. } c \cos. \text{co. } A$$

$$\text{or, } \sin. A = \cot. B \tan. b = \sin. c \sin. A$$

$$\sin. \text{co. } c = \tan. \text{co. } B \tan. \text{co. } A = \cos. a \cos. b$$

$$\text{or, } \cos. c = \cot. B \cot. A = \cos. a \cos. b$$

which serve for all possible cases.

LEGENDRE'S THEOREM FOR SOLVING TRIANGLES WHOSE SIDES ARE SMALL IN COMPARISON WITH THE RADIUS OF THE SPHERE.

If a small spherical triangle be described, and over its three sides we conceive threads to be stretched, the plane triangle whose sides are equal to the lengths of these threads will have for its angles (very approximately) the three angles of the spherical triangle, each diminished by one-third of its spherical excess.

Hence, if A and B be observed, and the included side c

measured, we determine first the area of the triangle considered as a plane one from the formula

$$\Delta = \frac{1}{2} c^2 \frac{\sin. A \sin. B}{\sin. (A + B)};$$

then the spherical excess

$$E'' = \frac{\Delta}{r^2 \sin. 1''};$$

then solve the plane triangle, whose angles are $A - \frac{1}{3} E$, $B - \frac{1}{3} E$, and included side c , and thus we have

$$a = c \frac{\sin. (A - \frac{1}{3} E)}{\sin. (A + B - \frac{2}{3} E)}$$

$$b = c \frac{\sin. (B - \frac{1}{3} E)}{\sin. (A + B - \frac{2}{3} E)}$$

$$c = 180^\circ + E - (A + B)$$

and thus the spherical triangle is completely determined.

FORMULÆ FOR GEODESIC MEASUREMENTS.

Various plans have been put in execution for determining the form and dimensions of the earth.

1. The actual admeasurement of degrees in various parts of the earth.

2. Pendulum observations, from which the force of gravity at different places is determined. This force is the resultant of the attraction of the earth's mass, and the centrifugal force arising from rotation. The centrifugal force in any latitude depends on the ellipticity and known quantities. The attractive force also depends on the ellipticity and law of density of the strata from the surface to the centre. This law is not absolutely known, but a highly probable one has been assigned. Also a relation of gravity in any latitude to equatorial gravity has been discovered, which is independent of any law of density, except that it continually increases from the surface to the centre. This was the discovery of Clairant, and is known as Clairant's formula. We are, therefore, enabled to express the force of gravity at any place in terms of the ellipticity, and by equating this with its observed amount, the ellipticity becomes known.

3. By observations on the moon. There are certain minute inequalities in the lunar motion caused by the oblate figure of the earth. The coefficients of the terms which represent these inequalities involve the ellipticity, and by comparing them with the values deduced from observation, we arrive at the ellipticity.

4. From the phenomena of precession and nutation limits between which the ellipticity must be included are deduced: from these it appears that the ellipticity must be greater than $\frac{1}{318}$, and less than $\frac{1}{315}$.

Of these methods the 2d and 3d are susceptible of the greatest accuracy, and the value of the compression deduced from each is $\frac{1}{308}$.

In the first method, which is that with which we are here especially concerned, the compression deduced from a great number of the best observations is $\frac{1}{317}$. (Vide Pontécoulant, *Sys. du Monde*, liv. v. chap. 6.) On the whole we may conclude that the compression is very nearly $\frac{1}{308}$; but from geodesic measurements we cannot be certain of this value within about its tenth part.

To deduce the compression from two meridional arcs, let m feet be the measure of 1'' of the meridian in latitude l .

$$m = A - B \cos. 2 l.$$

Suppose m_1, m_2 are two observed values of m , and l_1, l_2 the corresponding latitudes.

$$\begin{aligned} \text{Then,} \quad m_1 &= A - B \cos. 2 l_1 \\ m_2 &= A - B \cos. 2 l_2 \end{aligned}$$

$$\begin{aligned} \text{from which, } B &= \frac{m_2 - m_1}{\cos. 2 l_1 - \cos. 2 l_2} \\ A &= \frac{m_2 \cos. 2 l_1 - m_1 \cos. 2 l_2}{\cos. 2 l_1 - \cos. 2 l_2} \end{aligned}$$

Having thus determined A and B , we have

$$\text{Length of 1'' at equator} = A - B$$

$$\text{1'' at pole} = A + B$$

$$\therefore \frac{\text{Curvature at equator}}{\text{Curvature at pole}} = \frac{A + B}{A - B} = \frac{a}{b^2} \cdot \frac{a^2}{b} = \frac{a^3}{b^3}$$

where a and b are the equatorial and polar axes.

Hence $\frac{b}{a} = \left(\frac{A^2 - 1}{1 + B} \right)^{\frac{1}{2}} = 1 - \frac{2}{3} \cdot \frac{B}{A}$ nearly,
 since B is very small compared with A .

Hence compression $= 1 - \frac{b}{a} = \frac{2}{3} \cdot \frac{B}{A}$

Also, if ρ be radius of curvature at equator,

$$\rho \sin. 1'' = A - B = \frac{b^2}{a} \sin. 1''$$

$$\therefore b \left(1 - \frac{2}{3} \cdot \frac{B}{A} \right) \sin 1'' = A \left(1 - \frac{B}{A} \right)$$

$$A \left(1 - \frac{1}{3} \cdot \frac{B}{A} \right)$$

$$\therefore b = \frac{\sin. 1''}{\sin. 1''} \text{ the polar radius.}$$

$$A \left(1 + \frac{1}{3} \cdot \frac{B}{A} \right)$$

$$a = \frac{\sin. 1''}{\sin. 1''} \text{ the equatorial radius.}$$

Instead of depending on only two observations, it is usual to take several, and determine the average values of A and B by the method of least squares.

NUMERICAL EXAMPLE.

By Lambton's measures in India, the length of an arc of $1''$, in latitude $9^\circ 34' 44''$, is 100.78845 feet.

Also by Svanberg's measures in Sweden, the length of an arc of $1''$, in latitude $66^\circ 20' 10''$ is 101.63038 feet.

By taking the cosines of the double latitudes from a table of natural cosines, we have

$$101.63038 = A + B \times .677803$$

$$100.78845 = A - B \times .944619$$

$$\text{from which, } B = .49019$$

$$A = 101.27864$$

$$\text{and the compression } \frac{2}{3} \cdot \frac{B}{A} = \frac{1}{310} \text{ nearly.}$$

$$\text{Also equatorial radius in miles} = \frac{101.27864 \times 1.00161}{5280 \sin. 1''};$$

$$= 3962.83 \text{ very nearly.}$$

The above example is taken from Airy's Tracts, and the compression he deduces from it is $\frac{1}{306.3}$: but there must have been some error in his computation: in fact, the method he uses is less simple and more liable to error than the above. The above value of the equatorial radius is in accordance with the best determinations.

TO REDUCE AN ANGLE TO THE HORIZON.

When an observer is not provided with a theodolite, and wishes to ascertain the horizontal angle between two objects, if he observes with a sextant the two altitudes h and h' of the two objects, and the angle a between their summits, he may find the correction δa to be added to a in order to obtain the horizontal angle by the following.

Let h be greater than h' , and let each of them be expressed in seconds.

When $\frac{h - h'}{h + h'}$ is less than $\tan. \frac{a}{2}$, which will usually be the case, determine θ from the formula

$$\sin. \theta = \frac{h - h'}{h + h'} \cot. \frac{a}{2}.$$

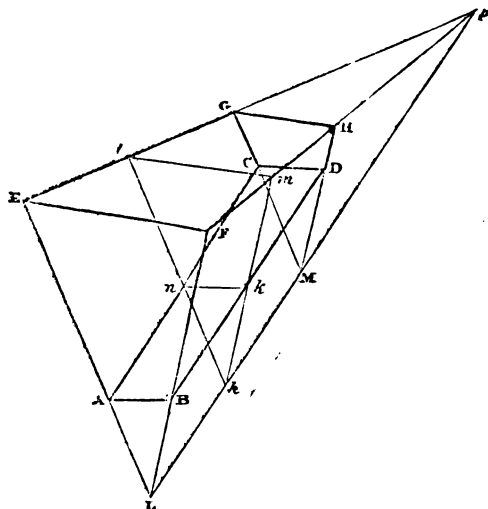
$$\text{Then } \delta a \text{ in seconds} = \sin. 1'' \frac{(h'' + h'')^2}{4} \tan. \frac{a}{2} \cos.^2 \theta.$$

If, however, $\frac{h - h'}{h + h'}$ be greater than $\tan. \frac{a}{2}$

$$\text{make } \sin. \theta = \frac{h + h'}{h - h'} \tan. \frac{a}{2}$$

$$\text{then } \delta a \text{ in seconds} = - \sin. 1'' \frac{(h'' - h'')^2}{4} \cot. \frac{a}{2} \cos.^2 \theta.$$

FORMULA FOR THE CONTENT OF EXCAVATIONS.



To find the solidity of a prismoidal excavation, $ABDCEF HG$, such that the transverse planes $EABF$, $GCDH$ are parallel, and at right angles to AC or BD , $ABCD$ being the road-way of uniform breadth; also that E, F, H, G are points in the same plane. Produce EA , FB to meet in L , also GC , HD to meet in M . Join LM . Produce EG , FH , LM , which, from the nature of the figure, necessarily meet in a point, P .

Let A be the area of the triangle ELF , a that of GMH , and α that of ABL or CMD . Also AC or $BD = LM = l$.

Content of pyramid $EFLP = \frac{1}{3} A \cdot LP$

$GMHP = \frac{1}{3} a \cdot MP$

\therefore Content of frustum $ELFGMH = \frac{1}{3} A \cdot LP - \frac{1}{3} a \cdot MP$

Also content of prism $ABLCMD = \alpha \cdot LM$

\therefore cont. of prismoid $EABFGCDH = \frac{1}{3} A \cdot LP - \frac{1}{3} a \cdot MP - \alpha \cdot LM$

$$\text{But } \sqrt{A} : \sqrt{a} :: LP : MP :: l + MP : MP$$

$$\therefore MP = \frac{l\sqrt{a}}{\sqrt{A} - \sqrt{a}}; \therefore LP = \frac{l\sqrt{A}}{\sqrt{A} - \sqrt{a}}$$

$$\text{Hence required content} = \frac{1}{3} \{ A + a + \sqrt{Aa} \} l - a l \quad (1).$$

Again, suppose EG bisected in l , and through l a plane to be drawn parallel to the transverse sections. Then $lnkm$ will be the mid-section. Let the area of triangle $lhm = m$. Then since its sides are respectively the arithmetic means of corresponding sides of ELF and GMH , and that the homologous sides are as the square roots of the areas.

$$\sqrt{m} = \frac{1}{2} \{ \sqrt{A} + \sqrt{a} \}$$

$$\text{Squaring } m = \frac{1}{4} \{ A + a + 2\sqrt{Aa} \}$$

$$\text{or, } \sqrt{Aa} = 2m - \frac{1}{2} (A + a)$$

Hence formula (1) becomes

$$\begin{aligned} & \frac{1}{3} \left\{ \frac{1}{2} (A + a) + 2m \right\} l - a l \\ &= \frac{1}{3} l \{ (A - a) + (a - a) + 4(m - a) \} \end{aligned}$$

But $A - a = \text{area } ABFE$, $a - a = \text{area } CDHG$,
and $m - a = \text{area } lnkm$.

Hence solidity of the excavation is

$$\frac{1}{3} AC \cdot \{ ABFE + CDHG + 4lnkm \},$$

which may be expressed as a rule, thus —

To the sum of the areas of extreme transverse sections, add four times the area of mid section, and multiply by one-sixth of the length of road-way, or distance between extreme sections; the product is the solidity required.

ON
SKETCHING GROUND
WITHOUT INSTRUMENTS.

THE great advantage of being able to form a plan with tolerable accuracy, without using instruments, is universally acknowledged ; and if large, some well understood arrangement should be made to connect together the several parts, particularly if performed by different persons.

It is proposed in this Essay to throw together such methods as are practised with the greatest facility or advantage, more systematically than in the former Treatise, where a single actual sketch in all its detail was exactly described.

As surveying and sketching are, in fact, nothing more than constructing imaginary right-lined figures on the ground, and similar ones upon paper, it is usual to employ instruments to measure angles, that we may overcome the obstacles constantly opposing our progress in right lines, or ascertain distances absolutely inaccessible. Hence, when a course of

surveying and sketching has been gone through, a facility of seeing the necessary figures, as well as the irregular figures of hills and other details, is obtained, and we become prepared for making sketches by the eye alone; not so exact as with instruments, yet very useful, when time will not permit greater accuracy.

Now, as the greatest of all our assistants, viz., angular measurement, is here taken away; having only pacing or judgment left to us, we must seek for some system by which these may be made subservient to the purpose, by examining the means left at our disposal of obtaining the wished-for object, and then indicate the method of employing them to the greatest advantage.

As all methods of surveying or sketching necessarily require the practical solution of triangles, or determining the relative position of three points with respect to each other, these being the most simple figures we can construct; we shall enumerate the most usual means of performing the problem in practice, and then show how they are applied to the proposed subject: we shall also endeavour to make them serve the double purpose of producing tolerable accuracy in large, as well as small detached sketches.

It may be premised that a right angle is the easiest constructed; it may be done by the eye alone; and that a right line can sometimes be produced to a considerable distance.

From these considerations it often happens, in accessible countries, that right lines may be paced,

and the perpendiculars upon them will give the contour of the ground ; also, that when obstacles arise, we can turn in either direction at right angles, and thus produce a sketch ; or even join many together, if each person employed on them starts from a known point in another's work ; which is, in fact, the *true* method of joining sketches together when *instruments* are used.

On other occasions three persons may pace on right lines, in like manner drawing the contour of hills, &c., by perpendicular offsets ; and then, having the three sides of a triangle, can lay their work together. Others may proceed to form triangles on these lines as bases, and so on.

When these lines are interrupted, we can go off at right angles, as before mentioned ; and sometimes regain the original line, if objects in that line have been before noticed as marks to guide

When neither method will suffice, we must have recourse to measuring angles, as hereafter shown, or to imaginary intersection, imitating the use of the plane-table.

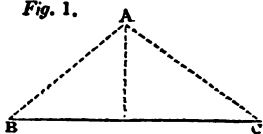
And when a country is so difficult of access as to prevent all these practices, we must absolutely judge the relative positions of principal objects, and fill in the intervening spaces by the eye alone.

We now proceed to recapitulate the various methods of solving triangles by pacing, and afterwards show their application by appropriate figures.

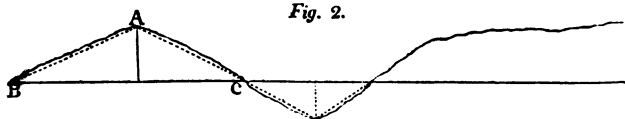
Let ABC in every case represent the angles, or, in practice, some objects situated at the angular

points. Now, if BC is paced, *Fig. 1.*

and the perpendicular upon it, the place of A , with respect to B and C , will be known. When many per-



pendiculars are used, this is the method by offsets; and several triangles, having their bases in the same line, give the contour of any irregular line whatever; thus:—

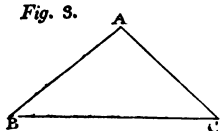


By an extension of this method in many planes, differently disposed, irregular solids, such as figures, ships, &c., are reduced to plans, and accordingly built like each other. It is hence used in making accurate models of every kind.

The other methods are precisely trigonometry practically worked, as the common instances may serve to show.

Let the three distances be all paced, and the place of A is determined as before. This is

seldom used in instrumental surveying, because, as in the following usual cases, we can dispense with one or two of the linear measurements, by taking angles instead.



Let BC be paced, the angles at B and C will determine the place of A ; or if BC be placed parallel to the original line upon the ground, and the observer being at A , takes the bearings back towards himself, he determines the place of A ; this is the method of interpolation. But having no instrument to take those angles, they may often be found thus:

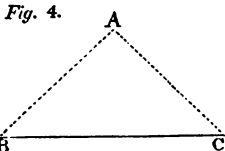
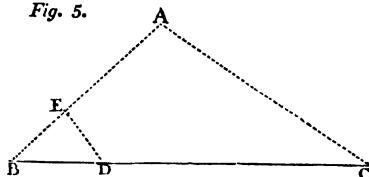
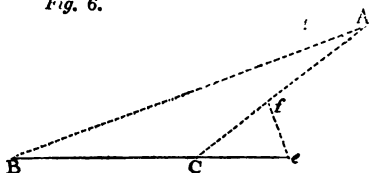


Fig. 5.



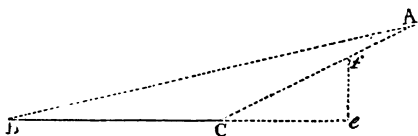
Upon BC pace BD , and upon BA pace BE , also ED ; then the angle B can be constructed as at *fig. 3.*; and in like manner the angle C can be found. If, as most frequently happens in roads, &c., the angle is very obtuse, proceed as follows:

Fig. 6.



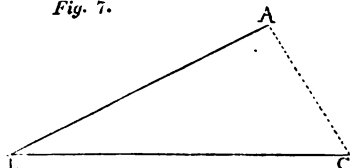
Pace forwards to e , and from C to f , also from f to e , which will give the supplement of the angle C , and is more correct than measuring its subtense. The angle B can be formed as in *fig. 5.*

Sometimes it is better done by a perpendicular, as in the following figure :



Again, we often pace two of the sides, and then get the contained angle, as in *fig. 5.*,

Fig. 7.



when AC becomes known.

These being of constant occurrence in practice, and the more complex problems seldom resorted to, they will not be mentioned ; but it may be observed, that when one triangle of great extent has been formed, any of these methods will form others on each of its sides, which are, in fact, bases to them, and these again to others, until it is necessary to verify them by some new pacing.

In going through a course of surveying and sketching, the pupil will be so familiar with what has been said, that it may appear unnecessary to enumerate the foregoing cases ; but it is better, perhaps, to bring them once more under his eye, that he may distinctly see the triangle is also the elementary figure in sketching without instruments.

The eye should be formed by using instruments at first, and when their use has been gradually dispensed with, not a single problem of the several that are in constant use will be impracticable, under certain limitations, in eye-sketching, as will be shown in the following instructions.

The pupil is supposed able to judge distances with tolerable precision, having had much occasion for that practice during the performance of his first surveys and sketches; and that he can depend upon his military paces sufficiently through a distance of one or two miles, for these paces are less fatiguing, and consequently more certain, than stepping yards; and they are easily convertible into yards, or reciprocally, by the following rules.

1st. Subtract from any number of military paces 1-6th of itself, and it will give the number of yards: thus,

6)2112 military paces.

— 352

1760 yards.

6)2976 paces.

— 496

2480 yards.

2nd. Add to any number of yards 1-5th of itself, and it will give the number of military paces: thus,

5)1760 yards.

+ 352

2112 military paces.

5)2480 yards.

+ 496

2976 paces.

If yards are preferred, it will make no other difference in the mode of proceeding than stepping each pace six inches longer.

The learner should also have practised judging longer distances than in surveying, such as tens, hundreds, and thousands of paces or yards; for, on some occasions, these longer measures alone can be used; and he should have verified his judgment by actually ascertaining distances assumed, to be assured of their correctness. Now, as no other linear measure is supposed to be in an officer's possession than what he can derive from pacing, or any other means of measuring angles than those already pointed out, or by imitating the use of a plane-table, and that he must, in many instances, depend upon his judgment alone for both, or even upon the paces of his horse, it follows that these means must be employed to the best advantage; and it must be confessed that much practice, with its resulting experience, can alone insure eminent success in this species of sketching.

To bring these elementary notions, before premised, to bear upon the subject in hand, we may consider two classes of eye-sketches necessary; and these naturally originate in the two great modifications of country in which sketches may be required.

First. — Hills of moderate elevation, or a comparatively flat country, of which a large reconnoissance may be wanted.

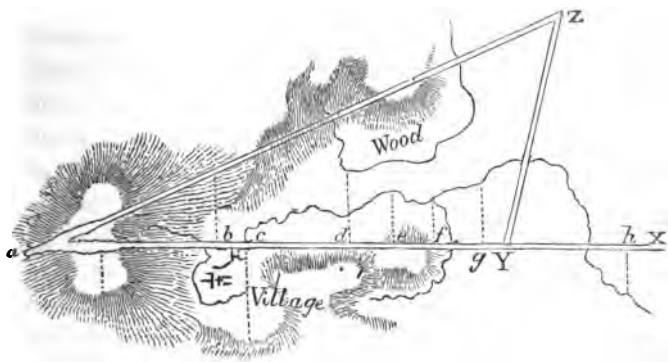
Second. — Mountains difficult of access, in which the passes, or small reconnoissances, may be sufficient.

In the first class almost every problem will occur.

The seven before mentioned will all be respectively useful; to exemplify which, we will first suppose the most simple proceeding sufficient, as in the following figure, by using one right line and its perpendiculars.

Suppose a road runs straight over a hill, and through a valley, as in the figure, it may be reconnoitred very easily, and drawn with tolerable perfection, as follows :

Fig. 8.

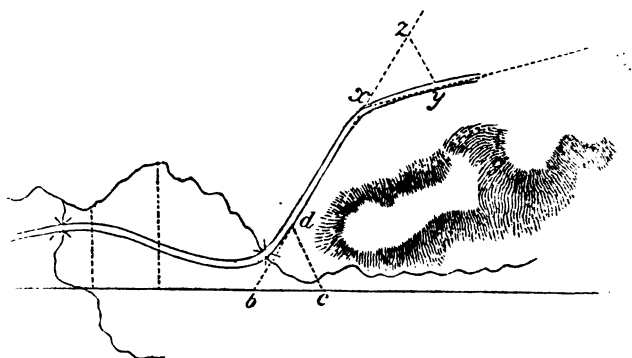


Pace the distance between *A* and *x*, noting at each place, as *a*, *b*, *c*, &c., the number of paces from *A*, and judging the perpendiculars, in short, every visible object of consequence, and the road turning off at *Y*.

If *Yz* and *Az* be paced, still more country can be drawn (see *fig. 3.*): moreover, if bad weather prevents drawing in the open air, or secrecy be ne-

cessary, a register of the several routes is easily kept, resembling a field-book, only with very few entries.

It has been already said that numerous obstacles will, in field practice, oppose us, and oblige us to turn in some other direction. The most common are, winding roads with hedges amongst low hills. It is, then, obvious that we can seldom operate by long right lines, without cutting through these obstacles, or having recourse to the measuring or judging of angles, as in *figs. 5, 6, and 7*. The following figure will illustrate this.



From *a* to *b*, offsets will give the road and stream: at *b*, we are in the line of the next bend of the road produced; the distance to the bridge can be paced or judged, the supplementary angle, *d b c*, measured, as in *fig. 6.*, and that at *x*, in the same way; while the hill, at *g*, can be judged and drawn between the

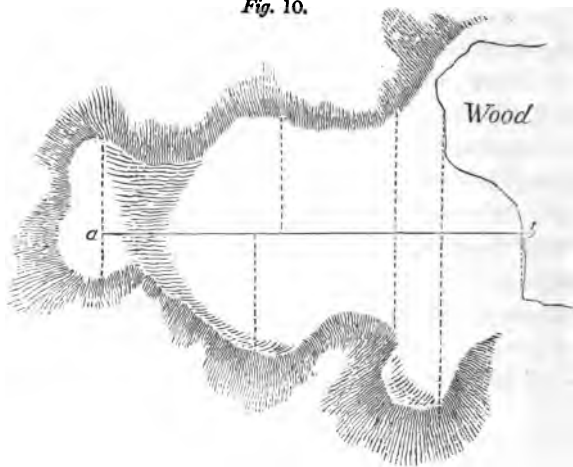
stream and road. While this method serves, only a little more trouble is given by winding roads; and we can, by what has already been taught, make rough plans, without any other instruments than a pencil and scale. Indeed, they provide for the occasional deviation from right lines effectually; and it is easy to make a plan of any ground chosen for an encampment expeditiously by them, as it will commonly be nearly level, as well as open. Crooked roads, when not fenced, are better drawn by right lines passing through or by them, as shown at *fig. 2*.

Another method of proceeding is drawn from the use of the plane-table, by imitating which we can readily intersect a distant point from two others known or assumed, if we place the line joining them on our sketch, as near as may be over the original line on the ground (*fig. 4.*), and in like manner the bends of roads, rivers, &c.; or, rather, the lines paced near them may be nearly ascertained, when surrounded by woods, hills, and so forth.

In those cases where the right line system can be acted upon, we may constantly turn off at right angles, and form a plan by this means, combining intersections, &c. with it as may seem best.

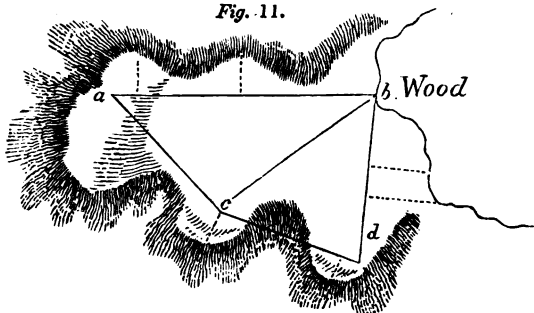
Let the annexed figure represent a piece of ground chosen for an encampment, the learner will immediately see how a plan can be formed from the lines drawn upon it:—

Fig. 10.



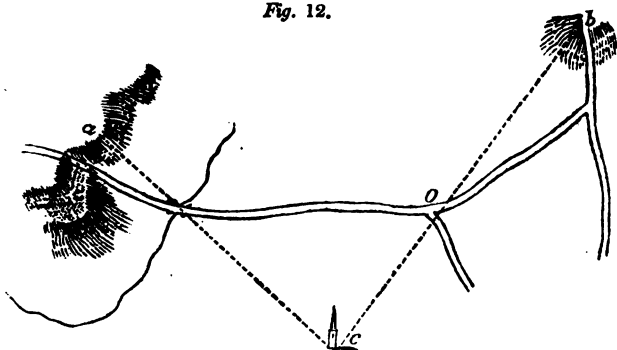
Again, if the offsets are too long, thus:

Fig. 11.



In both cases, whether by a single line, ab , and its perpendicular, or by the triangle abc , and bcd , with their perpendiculars, it is evident a plan may be formed from pacing, marking the distances upon a rough sketch on the ground.

It may be observed, that when a sketch is very forward, another source of assistance is opened to us; for it constantly happens, that two objects on the sketch, and in nature, can be brought into a right line on some part of which we stand: by moving backwards or forwards upon this line till two other objects, also on the ground and plan, are in a line, we find the place where we are. Also, lines may be produced through objects which will cut some place we desire to draw; and again, the same may be done from some other two, which will give it by intersection, as in the figure.

Fig. 12.

At *a* the signal tower *c* appears in line with the person and bridge; again, on the hill *b*, it appears exactly over the opening of the roads at *o*, and this determines its place. Many facilities of filling in a rough sketch will arise from these means.

It is presumed that sufficient means have been pointed out to produce hasty sketches with tolerable

correctness, some or all of which will, in various cases, be practicable. We must now consider the application of the same principle to more extended sketches.

Every three towns will clearly form a triangle; and as we have no instruments to measure their distances, we can only rely upon the estimated distances obtained by riding from one to the other, or local information. But these estimated distances will vary, often considerably, from the direct ones, because of the winding roads. It would be fruitless to attempt any exact scale of correction, for it depends upon so many variable circumstances, some of which may be mentioned here. The long post roads, on account of being more straight, will vary but few miles in one or two hundred; while those from town to town, for the same reason, will also sometimes vary inconsiderably; but the village roads, across the country, will often almost double the direct distance: these are all again modified by the nature of the country: it results, therefore, that to reconnoitre upon an extended space, a person should deduce, if possible, scales of correction for himself. With geographers, who draw so very small, a common and general supposition is, to reduce the direct distance by about 1-6th or 1-8th.

When, from observation and habit, experience has taught the probable correction, the reduced distances will form triangles approximating the truth, and the villages are placed upon the same principle, the whole of what has been before said will next supply the

means of filling in the interior rapidly ; and thus an approximate knowledge of a country is gained, which may serve until more correct surveys can be performed. In the filling up large spaces, division of labour must generally be resorted to, if expedition is desirable.

Now, by the methods before mentioned, several persons may perform their respective shares of the work, after the principal distances connecting the remoter objects have been laid down ; and whenever these methods fail, as they certainly will when the country is much enclosed, or at all intricate, the only remaining resource is, what may truly be called reconnoitring, for we can then only sketch the intermediate places from the top of some hill, church, tower, or any object from which the interior can be seen ; and however wild this notion may seem at first, yet the habit of doing so over small spaces of half or a quarter of a square mile, will certainly render it easy over much larger areas when necessary.

The general principle of joining many sketches or reconnoissances in one therefore requires that long lines should be ascertained in some manner near the truth, to serve as bases of operation ; and that each person's work should, if possible, be bounded by one of them at least ; that each should within his own boundary use the means before mentioned to perform his part of the work ; and that an arrangement as to the termination of these lines should be previously agreed upon before commencing it. In the hurry of war this may appear impracticable ; yet, as every

thing yields to perseverance, and as the means pointed out are abundantly sufficient, it must be hoped that this useful branch of art will, in the hands of British officers, assume some regular form when sufficiently practised.

The use of instruments has been studiously avoided in the preceding pages, although a common compass is sometimes very serviceable in keeping account of the bearings, and saves time. When an officer possesses one, he will of course use it at discretion.

There is yet another species of military reconnoissance remaining, when a hasty ride or walk over the ground is alone possible. It is evident that here a thorough command of the pencil and a well-practised eye are absolutely necessary, for every thing must be intrusted to memory, until a rude sketch can be drawn, which will evidently be more satisfactory than verbal description. It is thus that persons thoroughly conversant with plans are able to give a line of route to one another, by which the road to any particular place is most readily discovered, although to a perfect stranger, and yet no angle or distance can possibly be true, the whole coming directly from the mental impression remaining with the person who has furnished it.

The memoirs accompanying these reconnoissances should contain every particular of military or political importance which may fall within the officer's notice, or can be procured in his limited time. They should, in fact, be as replete with information as possible.

The column of route is not particularly specified, because the former rules and memoir before-mentioned, applied to a line of road extending many miles, constitute the very document in question.

In the second class, or mountain passes, much uncertainty must arise from the deception occasioned by the height of the surrounding hills, as the judged distances will greatly exceed their horizontal projections; nor will this be obviated by the use of any instrument that does not admit of either an approximate or proper reduction to the horizontal bases; such sketches must therefore be chiefly obtained by imaginary intersections of points perpendicularly over or under those which are to be drawn. The method of perpendiculars (*fig. 2.*) can seldom be resorted to, because of the impossibility of pacing the lines, on which, in lower hills, they are easily erected. It will therefore be necessary to assume, by mere judgment, the horizontal distances between some two elevated places, and by imaginary intersections to determine those of several others, drawing the contour of the intermediate features, with the roads, streams, &c., as near as this preparatory measure will permit. The reader will here remember what has been said before about filling in places from those which overlook them. If the road can be obtained first, in riding or walking up it, the surrounding mountains, with their under features, should be drawn while going along it, as at *fig. 8.* This will probably be most frequently the case, certainly whenever a pass is exceedingly difficult, and the

mountains among which it winds absolutely inaccessible. No instrument can possibly prepare a pupil for eye-sketches so well as a simple plane-table rightly used: he sees his plan gradually growing under his eye; he examines, verifies, and corrects, if necessary, the parts that may be defective; and the sketch possesses a spirit and fidelity which very few finished drawings from it preserve entire; besides, from habitually interpolating his place, he becomes so expert in judging the relative situations of himself with the surrounding objects, that it is at last quite easy, when an intricate place has been surrounded, by sketching as near as possible, to fill it in by mere examination; thus imperceptibly leading to one of the most useful practices in eye-sketching.

It results, therefore, from what has been said, that we must, in all cases, depend upon long lines, either exact or approximate, for bases of operation, where many sketches are to be joined in one; and that, in mountainous districts, we must work as near as possible like using the plane-table; and in this case little hopes can be entertained of joining many sketches tolerably, unless the points have been fixed trigonometrically.

The principles are drawn immediately from those of Geometry. The practice must necessarily be imperfect in a great degree, from the means employed being so. No rules can be given to suit all cases. They may be collected, as has been done in this short essay; but the officer being acquainted with them, must depend altogether upon his knowledge

and experience in their application. It might appear that the practical problems on the ground would be useful; and so they might, if they did not require time to perform, and an apparatus of poles and tapes, and calculation; but in this kind of sketching, no room is to be found for these operations, in places desirable as points, nor is the officer supposed to have any assistant with him, nor anything save his pencil, scale, paper, and drawing-case. Yet, in the preparatory business of ascertaining long distances, they may be useful, if time can be found for employing them.

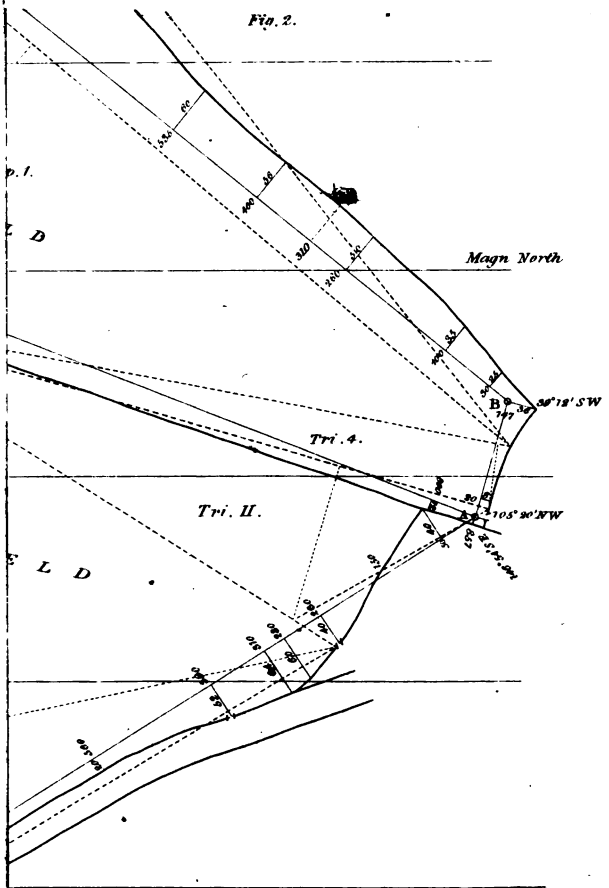
The following Table will exhibit an arrangement of some of the most useful information which should accompany the Sketches, and must be varied according to existing circumstances.

Nature of Soil.	Roads.	Woods and Plantations.	Water.	Cultivated or Pasture.	Angle of principal Slopes.	Other Remarks.
Chalk, Limestone, Gravel, and Clay,	Principal Roads good, inferior Roads almost impassable in winter.	Oaks and Elms; much underwood.	Streams good, clear, wholesome, and abundant.	Well cultivated, good Pasturage.	15° to 25°.	



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Fig. 2.





Corpⁿ

AL CITY

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in march

uggons

other Military details will be found in works on
 Fortification to which they especially belong
 Troops, in the instructions lately published by the
 Adjutant General



Foot Bridge



Battle fought



br Sect



Windmills



Water Mill



Small forts & battery



Troops dark line in front



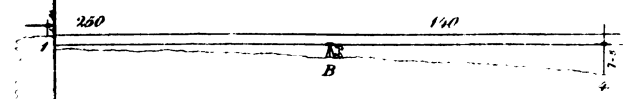
Videttes



Chevaux de frize



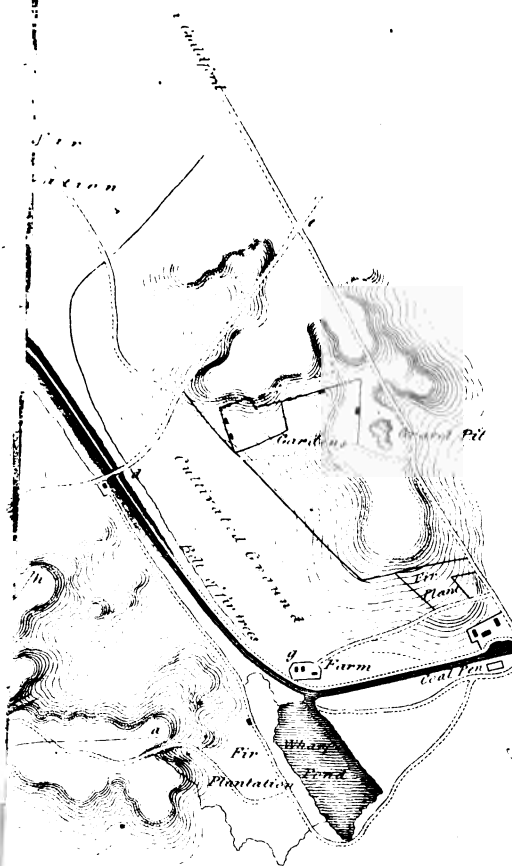
Palisades



G.D.

T. Bradley 1862

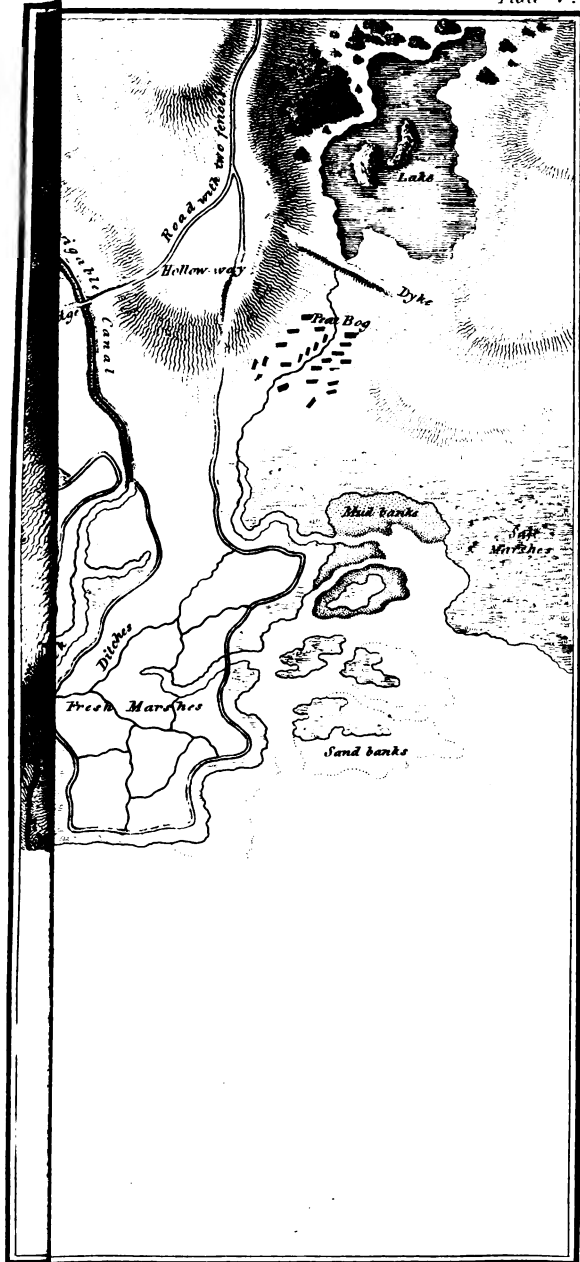




Remarks The whole is heath except the part marked cultivated.
The canal runs through a hollow way about 70 feet deep.

T. Bradley s.







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